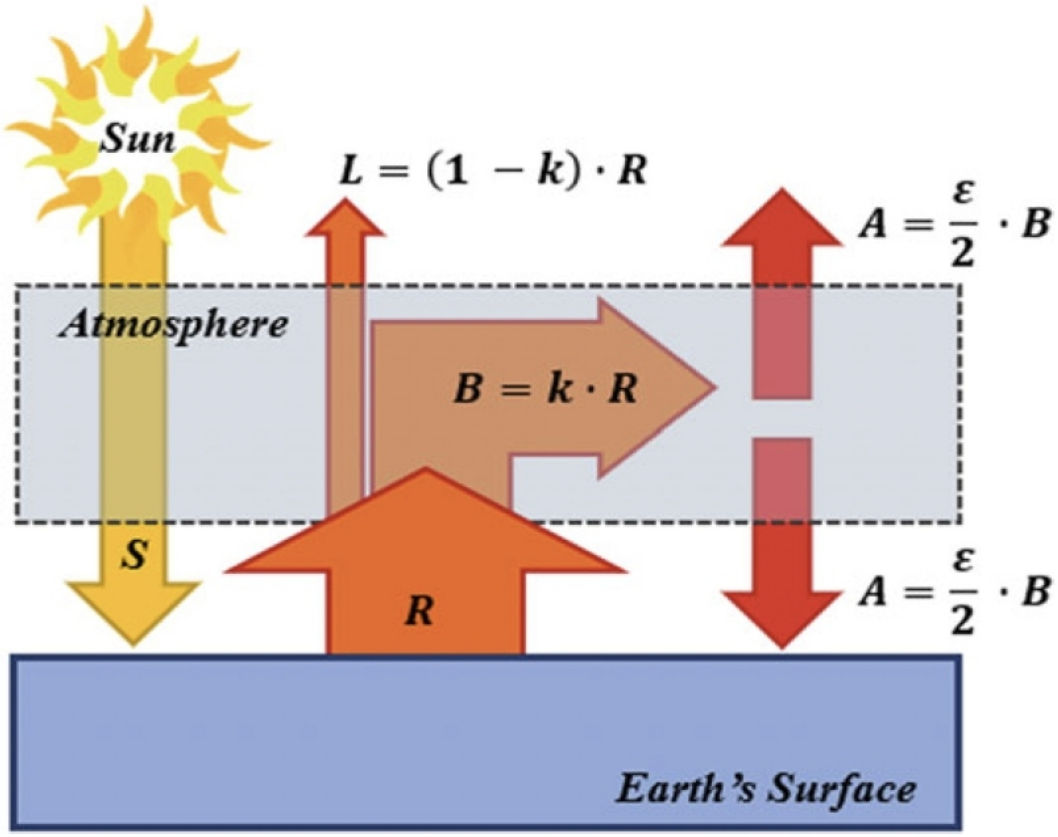
BS

QUANTITATIVE REASONING-I

Course Code: 9424/9836



Department of Mathematics
Faculty of Sciences
ALLAMA IQBAL OPEN UNIVERSITY

QUANTITATIVE REASONING-I

Course Code: 9424/9836

Units: 1–9



DEPARTMENT OF MATHEMATICS
ALLAMA IQBAL OPEN UNIVERSITY
ISLAMABAD

COURSE DEVELOPMENT TEAM

Chairperson Course Team:	Prof. Dr. Hajra Ahmed
Course Coordinator:	Dr. Rizwan Salim Badar
Unit Writers:	Dr. Muhammad Nazam
	Dr. Irfan Younas
	Dr. Bismah Jamil
	Dr. Rizwan Salim Badar
Units Reviewer:	Dr. Muhammad Asad Zaigham
Editor:	Mr. Fazal Karim
Typeset / Layout:	M. Hameed Zahid

FOREWORD

This study guide has been designed to teach our students how to use quantitative reasoning, logical thinking, and basic mathematical skills to solve problems in real-life situations. The purpose is to assist the students in developing the capacity to provide thoughtful and fact-based arguments. To demonstrate the importance of quantitative reasoning in all areas of learning thought-provoking questions have been included from a variety of academic and real-world subjects.

Quantitative Reasoning has been a compulsory topic now in syllabi of all universities as recommended by Higher Education Commission of Pakistan. It acts as a thorough manual, offering the fundamental information required to understand quantitative reasoning and its real-world applications. It is created in a performance-based manner with the goal of imparting fundamental concepts in mathematics. This novel course format has been shaped by countless hours of hard work, dedication and fruitful conversations. It is our firm belief that this guide will satisfy students' intuitions and offer them visual confirmation. This text places a higher priority on conceptual comprehension and practical application than traditional methods, which place greater value on formal mathematical proofs.

Finally, I would like to express my sincere gratitude to everyone who has contributed, no matter how tiny, to make this guide a reality in such a short period of time. I sincerely appreciate all the hard work and continued efforts for timely completion of the task. We have worked together to produce something genuinely valuable, and I hope that it will inspire students to improve their learning abilities and logical thinking.

Prof. Dr. Nasir Mahmood
(Vice-Chancellor)

MESSAGE FROM THE DEAN

The scientific discipline that establishes connections and bridges the gaps across the natural sciences is mathematics. Though it appears rather abstract by nature, mathematics is used in other sciences. Its uses immediately become apparent to us. For our pupils to internalize the foundational ideas, this course is extremely necessary.

This guide covers a lot of the foundational mathematics required for business and other social science courses. I would want to sincerely thank, appreciate, and congratulate everyone who helped in the preparation of this book. This will undoubtedly help our pupils refine their business concepts quantitatively.

Prof. Dr. Hajra Ahmed
Dean, Faculty of Sciences

ACKNOWLEDGEMENTS

I am grateful to God Almighty for giving me the opportunity to supervise the development of the course 'Quantitative Reasoning'. I would like to express my gratitude to my worthy colleagues who contributed to writing units, proofreading, editing and designing this guide.

I would like to thank Dr. Nasir Rehman and Dr. Irfan Mustafa for their constant support and encouragement. I am especially grateful to Dr. Asad Zaigham for reviewing the manuscript of the book in detail.

I would like to thank writers Dr. Muhammad Nazam, Dr. Irfan Yunus, Dr. Bismah Jamil and Dr. Rizwan Salim Badar for writing this course on a priority basis. Particular thanks go to Dr. Rizwan Salim Badar and Mr. Mujahid Ahmad for proofreading and composing the guide.

I am highly grateful to Prof. Dr. Nasir Mahmood, Vice-Chancellor AIOU and Prof. Dr. Hajra Ahmed, Dean Faculty of Sciences AIOU for giving a conducive environment to complete this course in the shortest possible time. I hope that the contents of the book would help the students in developing the critical thinking which would help them in solving real world problems. Suggestions and criticism to improve the text will be highly appreciated. It can be forwarded at iqbal.zahid@aiou.edu.pk

(Dr. Zahid Iqbal)
Associate Professor/Chairman
Department of Mathematics AIOU

COURSE INTRODUCTION

This course is designed to enhance your analytical and critical thinking skills through the application of mathematical concepts. In this class, you will explore a range of topics including logic, probability, statistics, and mathematical modeling, all aimed at developing your ability to reason quantitatively and solve real-world problems. By engaging with practical examples and hands-on activities, you will learn to interpret data, make informed decisions, and communicate your findings effectively. This course will not only improve your mathematical proficiency, but also empower you to approach complex issues with confidence and clarity.

OBJECTIVES

The objectives of the Quantitative Reasoning course are to:

- Develop critical thinking and problem-solving skills using mathematical concepts.
- Understand and apply principles of logic and reasoning to analyze arguments and claims.
- Enhance proficiency in interpreting and analyzing data through statistical methods.
- Learn to use probability to assess risk and make informed decisions.
- Apply mathematical modeling to represent and solve real-world problems.
- Improve the ability to communicate quantitative information clearly and effectively.
- Foster the capability to use quantitative reasoning in everyday life and professional contexts.

HOW IS THE COURSE ORGANIZED?

The guide is divided into nine units in one volume. An attempt has been made to present the material in an informal way. Only those topics of Mathematics are covered which are thoughtful to be useful for everyone, e.g., we started studying Mathematics when we were in grade one and we continue its studies. In spite of this, most of us do not have full command of everyday arithmetic.

Unit-1: This unit is focused on the study of the importance of quantitative reasoning skills. Quantitative reasoning, or quantitative literacy, refers to a key set of skills that students need to possess to draw meaningful conclusions and make informed decisions based on numerical information. In this unit, we discuss the importance of quantitative reasoning skills, learn about the history of Mathematics, the contributions of Muslim scholars in the advancement of Mathematics and understand the concept of several types of numbers and their applications in daily life problems.

Unit-2: In this unit, we discuss problem solving techniques, firstly we will be defining parts and whole and their relationship. In the context of this, examples from real life will be discussed. Secondly, we would discuss units and rates and see their applications from real life scenarios. The questions related to the above-mentioned topics will be given at the end.

Unit-3: In this unit, we will explore the relationships between numbers, the building blocks of mathematics and the universe. We discuss sets of numbers and their types, understanding big and small numbers. At the end of this unit, we introduce mathematical uncertainty and its applications such as finance and risk management, heath care and making decisions, artificial intelligence and machine learning.

Unit-4: In this unit, we discuss 'Financial Issues,' which provides an in-depth examination of various financial difficulties. We explore basic theories and methods, assess a wide range of real-world situations, and provide practical insights and solutions for solving financial challenges with knowledge and understanding by exploring the complex nature of financial phenomena.

Unit-5: In this unit, we introduce Mathematical expressions that cover the basic ideas behind mathematical functions and how they are represented graphically. Equating two expressions in one variable and using it to solve practical problems. Also, we discussed the Golden Ratio and how it affects the arts and design. Finally,

we will talk about numerical patterns and how they can be used for prediction, problem solving and their applications.

Unit-6: In this unit, we learn about geometrical objects through architecture and landscape dealing with social and economic issues involving geometrical objects. Dealing with social and economic disciplines, we try to understand the use of notions of symmetry and congruence in geometrical shapes.

Unit-7: The focus of unit-7 is on the Venn Diagrams. This unit contains the definition of the Set, how to compute the domains and their ranges. Furthermore, some practical scenarios are presented to understand the relationship between Sets and Venn Diagrams. At the end, some self-assessments questions of Venn Diagrams are given.

Unit-8: In this unit, the idea of Mathematical Inequalities is presented. We discussed several practical problems involving inequalities in different disciplines. At the end of the unit, the students will be able to introduce absolute values and inequalities.

Unit-9: This unit studies the idea of Logical Thinking. In essence, logical thinking refers to the practice of sound reasoning or making good arguments. Logical thinkers observe, analyze, and scrutinize phenomena, reactions, and feedback before drawing conclusions based on the information they have collected. Exploring real-world applications of logic across different fields and scenarios. At the end of this unit, we discuss some self-assessment questions.

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Unit–1

IMPORTANCE OF QUANTITATIVE REASONING SKILLS

Written by: Dr. Irfan Younas

Reviewed by: Dr. Muhammad Asad Zaigham

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1.1 Introduction

In the modern world, it is necessary to possess the skill of quantitative reasoning. This ability enables people to think clearly and make informed decisions based on numerical data, which is essential in almost all academic fields. Moreover, quantitative reasoning is used extensively in various professions and is necessary for people to make sound decisions in their daily lives. In essence, the ability to reason quantitatively is a fundamental aspect of being a well-informed and capable individual in modern society.

1.2 Objectives

At the end of the unit (1), you will be able to:

- Understand the importance of quantitative reasoning skills
- Know about the history of Mathematics
- know about the contributions of Muslim scholars in the advancement of Mathematics
- understand the concept of different types of numbers and their applications in daily life problems.

1.3 Importance of Quantitative Skills

Quantitative reasoning, or quantitative literacy, refers to a key set of skills that students need to possess in order to draw meaningful conclusions and make informed decisions based on numerical information. This involves the ability to analyze and interpret data, identify trends and patterns, and use mathematical models to make predictions and draw conclusions. Students who are proficient in quantitative reasoning are able to construct persuasive arguments that are supported by quantitative evidence and can communicate these arguments clearly and effectively using a range of different formats, such as tables, graphs, charts, and equations. They are also able to apply their skills in a variety of different contexts, from academic research to professional settings, and are well-equipped to succeed in a rapidly changing and data-driven world.

Quantitative reasoning (QR) skills are a set of abilities that enable individuals to approach theoretical questions with data and numerical evidence. It involves basic math skills, such as algebra, that are used to investigate and interpret real-world quantitative data. QR skills help individuals to draw relevant conclusions that can be applied to their daily lives. These skills are not limited to any particular discipline and can be applied across a range of interdisciplinary problems. They are particularly useful in fields such as economics, science, and engineering where quantitative data analysis is critical to making informed decisions.

Teaching quantitative reasoning (QR) is crucial in helping students develop the ability to analyze and interpret data, think critically, and apply basic mathematical skills to solve complex problems within and across different academic disciplines. By teaching QR, students can learn how to evaluate and draw conclusions from data, identify patterns, and make informed decisions based on evidence. This skill is essential not only for academic success but also for real-world applications in fields such as business, science, and technology.

At Allama Iqbal Open University (AIOU), we believe that students should have the necessary skills to become proficient in mathematics. Our goal is to equip them with the skills they need to succeed in their professional, personal, and civic lives. We want to ensure that students are well-versed in quantitative reasoning (QR) and numerical literacy. To achieve this, AIOU is committed to integrating numerical literacy throughout all curriculums. The Quantitative Reasoning (QR) courses are an essential part of the Pathways Student Learning Outcomes (SLOs). These courses are designed to provide students with the foundational knowledge and skills they need to excel in their academic pursuits. The SLOs are tailored to meet the needs of students across different academic disciplines. They will help students develop the quantitative reasoning (QR) skills they need to succeed in their

professional and personal lives and to fulfill their civic responsibilities. We believe that the integration of numerical literacy throughout all curriculums will not only help students to excel in their academic pursuits, but also prepare them for future challenges. At AIOU, we are committed to providing students with the tools they need to succeed in a rapidly changing world.

1.4 History of Mathematics

1.4.1 Pre-Historic

Mathematical thinking has evolved from the earliest concepts of numbers, magnitude, and patterns in nature. Recent studies have shown that these ideas are not unique to humans. They were probably an essential part of everyday life in early hunting and gathering societies. The concept of numbers evolved gradually over time, as evidenced by languages that distinguish between "one", "two", and "many", but have no words for higher numbers. This gradual evolution of numerical concepts suggests that early humans may have used counting strategies to keep track of the animals they hunted or the objects they gathered.

The Ishango bone, found in northeastern Congo, is over 20,000 years. It is regarded as the oldest tool of the mankind. There are a few marks carved in three columns. It is believed to be either a tally of prime numbers or a six-month lunar calendar. Its influence on the development of mathematics in Egypt is disputed.

Egyptians used geometric designs in the 5th millennium BC. Some megalithic structures in England/Scotland, dated 3rd millennium BC, may have geometric concepts. These claims are disputed. The Babylonian and Egyptian documents are considered as the oldest undisputed documents in the mathematical history.

1.4.2 Babylonian

Babylonian mathematics is the mathematics that evolved over the centuries and was developed by the people of Mesopotamia, which is modern-day Iraq, from the early Sumerian era. Their works have been studied and analyzed by scholars over the years and have been found to be an essential contribution to the development of mathematics. It includes a wide range of topics such as geometry, algebra, and arithmetic. The Babylonians were renowned for their advanced knowledge of mathematics in their time. The importance of Babylon as a centre of study is why it is called Babylonian mathematics. During the Arab Empire, Mesopotamia, particularly the city of Baghdad, became once again a significant hub for the study of Islamic mathematics.

Babylonian mathematics is known to us by the over 400 clay tablets inscribed in Cuneiform script, unlike Egyptian mathematics which has very few sources. The

Sumerians, around 2500 BC, wrote multiplication tables, and geometrical and division problems on clay tablets, which also saw the earliest numerals of Babylonians.



Figure 1. 1 Geometry Problem on a Susa Clay Tablet from
2nd Millennium BCE Scribe School

The Babylonians achieved a remarkable achievement in the field of mathematics. The introduced a sexagesimal number system, which was based on 60 and allowed for easy calculations by hand. This system is still used today in the form of 60 seconds in a minute, 60 minutes in an hour, and 360 degrees in a circle. The Sumerian scribes likely adopted this system because it could be evenly divided by many numbers, making it practical for doling out grain allotments. However, it's also possible that it was an ethno-linguistic phenomenon. What set the Babylonians apart from the Egyptians, Greeks, and Romans was their use of a place-value system. Unlike other civilizations, the Babylonians could write fractions as efficiently as whole numbers, giving them remarkable computational accuracy. They didn't have a decimal point, but they used a zero symbol as a placeholder for empty positions. However, they never develop a true place value system.

1.4.3 Egyptian

Egyptian mathematics includes mathematical writings in the Egyptian language, which later transitioned to Greek and Arabic. The numeral system of ancient Egyptians has roots in Sub-Saharan Africa and has been linked to fractal geometry designs in Egyptian architecture. The Rhind Papyrus, dating back to 1650 BC, is a significant Egyptian mathematical text that provides instructions for arithmetic and geometry and contains evidence of various mathematical knowledge. The Moscow Papyrus, from around 1890 BC, contains word problems, including a technique for calculating the volume of a frustum. The Berlin Papyrus, from around 1800 BC provides evidence of the ancient Egyptians' ability to solve second-order algebraic equations.

1.4.4 Greek

Greek mathematics, developed by Greek mathematicians between 600 BC and 529 AD, was more advanced than previous mathematics. Babylonian and Egyptian mathematicians used inductive logic to draw certain conclusions in mathematics, while Greek mathematicians laid the foundation of deductive logic, i.e., using logic to draw conclusions from axioms, definitions, and rigor mathematical calculations.

The origins of Greek mathematics can be related to the works of Thales of Miletus and Pythagoras of Samos. It is believed that these two great mathematicians were heavily influenced by the mathematical knowledge of the ancient Egyptians and Babylonians. A legendary mathematician, Pythagoras even learned the principles of astronomy, mathematics and geometry from the Egyptian priests.

Thales was a pioneer in solving geometry problems, such as computing the height of pyramids and the distance between ships and from the shore. He is renowned for using deductive logical reasoning in geometry problems. He derived four corollaries to Thales' Theorem. He is considered the first mathematician credited with a mathematical discovery, having derived four corollaries to Thales' Theorem. Pythagoras, on the other hand, the founder of the Pythagorean School, postulated that mathematics governs the universe. The Pythagoreans, his followers, coined the term "mathematics" and embarked on studying mathematics for its intrinsic value. Pythagoras is renowned for his contribution to mathematics, which includes the first proof of the Pythagorean theorem, a proposition in geometry that states that the square of the hypotenuse (the side opposite the right angle) of a right-angled triangle is equal to the sum of the squares of the other two sides. Additionally, he is credited with proving that irrational numbers exist, which are numbers that cannot be expressed as a ratio of two integers. These contributions have had a profound impact on the field of mathematics and continue to inspire mathematicians and scientists to this day.

Plato, a renowned philosopher, and mathematician who lived from 428/427 BC to 348/347 BC, played a significant role in guiding others about the beauty of mathematics. In the 4th century, Plato's Academy, situated in Athens, evolved as the hub of mathematical learning, and famous mathematicians of the day, such as Eudoxus of Cnidus (c. 390 - c. 340 BC) and Aristotle (384–c. 322 BC) emerged from there. Plato contributed extensively to the basis of mathematics by clarifying the definitions and assumptions such as that of a line as "breadthless length". He also developed a method for finding Pythagorean triples, which are still used today. Eudoxus, a student of Plato, made significant contributions to mathematics by developing the method of exhaustion. Aristotle (384–c. 322 BC), was a student of Plato and one of the most influential philosophers in Western thought. He significantly contributed to mathematics by developing the basis of modern

logic. These foundations have since become the standard for logical reasoning and have been extensively studied by mathematicians and philosophers alike.

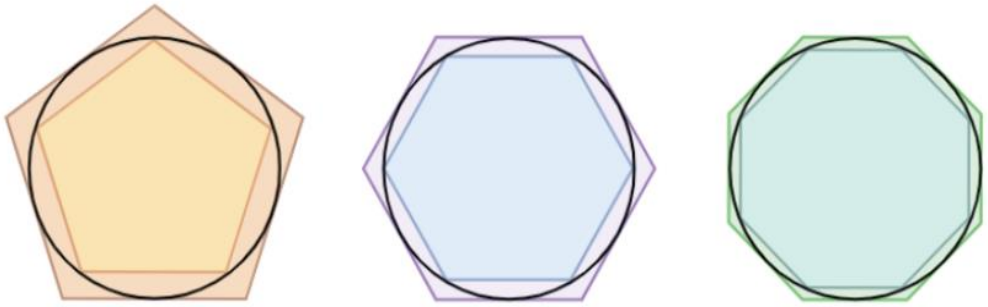


Figure 1. 2 Archimedes Used the Method of Exhaustion to Approximate the Value of pi.

Many people think that Archimedes, who lived in Syracuse from about 287 BC to 212 BC, was the smartest scientist in ancient times. The way he used it to figure out the area under the path of a parabola was one of his most important contributions. He did this by adding up an infinite series, which is a lot like how calculus is done today. In addition, Archimedes found a way to get the value of π as accurately as needed. At that time, the most exact figure of π that was known to him was between 3+10/71 and 3+10/70.

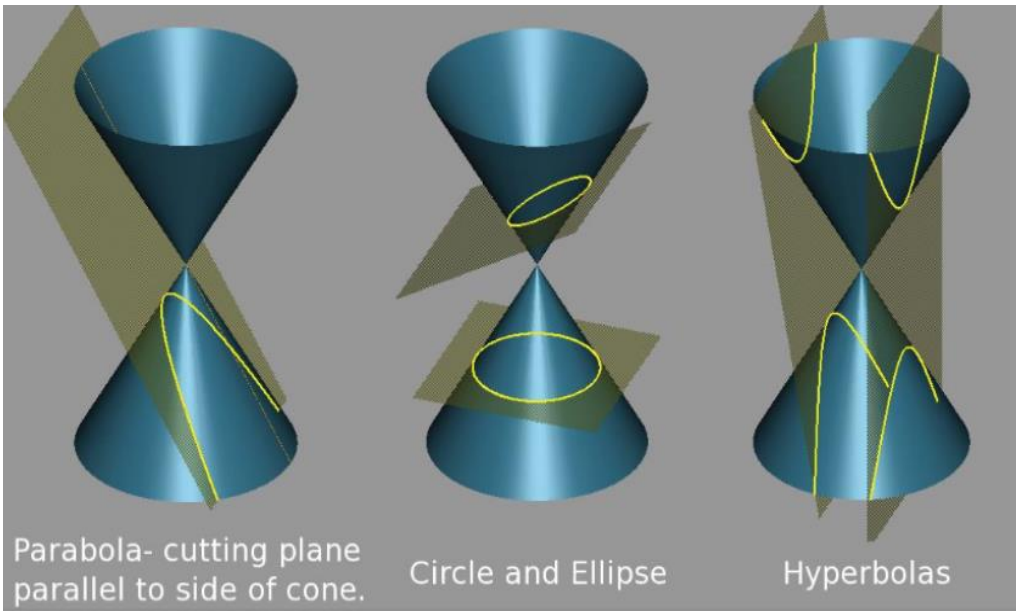


Figure 1. 3 A lot of Progress was made in the Study of Conic Sections by Apollonius of Perga

Apollonius of Perga was a famous scientist who lived from 262 BC to 190 BC. He made important contributions to the study of conic sections. He showed that the angle of the plane that cuts a double-napped cone can be changed to get all three types of conic sections. Apollonius also named conic sections "parabola," "ellipse," and "hyperbola." The word "parabola" means "place next to" or "comparison," and the word "ellipse" means "deficiency." This gave rise to the language used for conic sections today. Among his many contributions, Apollonius authored the well-known and preserved mathematical work "Conics".

The Almagest, a Greek astronomical treatise by Claudius Ptolemy, is considered one of the most important works of antiquity. It is a comprehensive work on astronomy and includes a vast amount of information on the movements of celestial bodies, as well as detailed trigonometric tables. Ptolemy's work revolutionized astronomy, and his contributions to trigonometry are equally significant. The trigonometric tables in the Almagest were used by astronomers for over a thousand years, and they are still studied today. These tables contain values for the sine, cosine, and tangent functions, as well as chords and arcs. They were used to calculate the positions of planets and stars and are still used in modern astronomy. Ptolemy's theorem, which he is credited with, is a theorem used to derive trigonometric quantities. The theorem is still widely used in geometry and trigonometry today. His work on trigonometry and mathematics was groundbreaking, and his contributions continue to influence fields such as astronomy, physics, and engineering.

Hypatia of Alexandria, born in AD 350, was a remarkable woman and the first recorded female mathematician in history. Her father had been the Librarian at the Great Library of Alexandria, and she took over as head librarian after him. Her work in applied mathematics was very well known and respected, and many of the books she wrote on the subject are still studied today. Sadly, Hypatia's life was cut short because of a political fight. The Christians in Alexandria didn't agree with her lessons and views, so they stripped her in public and put her to death when she was only 65 years old. It was sad that she died, and many people think that it was the end of the era of Alexandrian Greek mathematics. However, despite this setback, the work of Greek mathematics continued in Athens for nearly a century under the guidance of other influential figures such as Proclus, Simplicius, and Eutocius. Despite the loss of Hypatia, her contributions to mathematics and philosophy have continued to be celebrated and her legacy has inspired generations of women to pursue careers in STEM fields.

1.4.5 Roman

Ethnic Greek mathematicians continued their work during the Roman Empire. Romans were skilled in applying mathematics, detecting financial fraud and managing taxes. Romans used mathematics to resolve issues in engineering,

including art and the construction of architecture. They created a Roman calendar using basic mathematics, with the Julian calendar including a leap day every four years. The Gregorian calendar corrected the Julian calendar's 11-minutes-and-14-second error and is now the international standard calendar. The Han Chinese and the Romans developed a device called the wheeled odometer at around the same time. This device was used to measure distances travelled.

1.4.6 Chinese

In 212 BC, Emperor Qin Shi Huang ordered all books in the Qin Empire to be burned, with the exception of those with official approval. Even if not, everyone followed it, this structure resulted in a lack of prior knowledge of ancient Chinese mathematics. However, the Han dynasty (202 BC–220 AD) produced mathematical works that increased the understanding of the previous time after the book burning in 212 BC. "The Nine Chapters on Mathematical Art," which had 246-word problems on a range of topics including surveying, geometry, business, agriculture, and engineering, was the most significant of these publications.

1.4.7 Indian

The Indus Valley civilization, which existed in the Indus River basin, is a remarkable example of early civilization in India. The cities of this civilization were notable for their geometrically regular layout, although no mathematical documents have survived from this period. The Sulba Sutras are fascinating records of early Indian mathematics, dating back to various periods between the 8th century BC and the 2nd century AD.

Pingala, an ancient scholar, developed a method for counting the syllables in poetry that is similar to the way computers use binary code. He also explored the different ways that syllables can be combined to create different patterns, which is similar to the binomial theorem. In addition, Pingala discovered a sequence of numbers that we now call Fibonacci numbers, which he referred to as "mātrāmeru."

Madhava of Sangamagrama in the 14th century discovered several mathematical series, including one that was used to estimate the value of π. In the 16^{th} century, the Kerala School's advances in classical analysis may have influenced European developments in calculus via Jesuit missionaries and traders. However, some experts question whether the Kerala School created a clear theory of differentiation and integration, or if they simply shared their findings with others beyond Kerala.

1.4.8 Contributions of Islamic Empires

Over the course of the eighth century, the Islamic Empire expanded to include parts of India, the Middle East, Central Asia, North Africa, and Iberia. Among the many things this kingdom accomplished was the advancement of mathematics. During this period, Arabic was the primary language used to write the majority of

mathematical literature. It is crucial to keep in mind, nevertheless, that not all of the authors of these works were Arabs. A famous Persian scientist named Muḥammad ibn Mūsā al-Khwārizmī wrote two important books in the ninth century. One was about Hindu-Arabic numbers, and the other was about how to solve equations. The book "On the Calculation with Hindu Numerals," which was written by Al-Khwārizmī in 825, brought Indian numbers and math to the West. The works of Al-Kindi and Al-Khwārizmī were the first to bring algebra and complicated math problems to the West. The word "algebra" comes from the title of al-Khwārizmī's book, "Al-Kitāb al-mukhtaṣar fī hīsāb al-Ėabr wa'l-muqābala." The word "algorithm" comes from "Algoritmi," which is the Latinized form of al-Khwārizmī's name.

The first individual to teach algebra as a distinct topic in elementary form was Al-Khwārizmī. His algebra differed from the earlier mathematical algebra, which played a crucial role in resolving a series of problems. In contrast, al-Khwārizmī's algebra was an exposition that started with primitive terms, with the combinations capable of giving all possible cases for equations. Al-Khwārizmī investigated equations for their own sake and in a generic way that could approach an indefinite class of mathematical problems.

Abu Kamil, a renowned mathematician from Egypt, made significant contributions to the field of algebra during the 9th century. One of his major achievements was extending algebra to include irrational numbers. At that time, it was a very new idea to think about getting the square and fourth roots as answers to quadratic problems. One of Abu Kamil's other amazing achievements was coming up with ways to solve three nonlinear problems with three factors at the same time. He also tried to find all the possible answers to some of his problems, which led him to find 2676 answers for one of them. This was another important part of his work. Abu Kamil made significant contributions to mathematics that continue to inspire discoveries today.

1.4.9 Renaissance

During the Renaissance, math and accounting were connected. Children of merchants were taught these subjects together in reckoning or abacus schools to learn skills useful for trade and commerce.

Trigonometry emerged as a significant branch of mathematics driven by the increasing demand for accurate maps and navigation. In 1595, Bartholomaeus Pitiscus used the word "Trigonometria" in his publication. Earlier, in 1533, Regiomontanus published a table of sines and cosines.

1.5 Mathematics Development in Scientific Revolution

1.5.1 17th Century

The 17th century was a time of great progress and innovation in the fields of mathematics and science. Visionaries like Galileo, Tycho Brahe, Johannes Kepler, Isaac Newton, and Gottfried Wilhelm Leibniz made groundbreaking discoveries that laid the foundation for many of the scientific and technological advances we enjoy today. Their work expanded our understanding of the universe and paved the way for future generations to build upon their achievements. Additionally, their application of mathematics to new areas, such as probability theory and combinatorics, opened up exciting new avenues of research and innovation. Overall, the 17th century was a pivotal moment in the history of science and mathematics, and its impact can still be felt today.

1.5.2 18th Century

A number of mathematicians made important contributions to the area in the eighteenth century. But because of his numerous contributions to the fields of calculus, complex analysis, combinatorics, and topology, Leonhard Euler is regarded as the most important. In addition to initiating the field of graph theory studies, he standardized a number of contemporary mathematical notations. Joseph Louis Lagrange and Pierre-Simon Laplace are two more well-known mathematicians from the 18th century who made groundbreaking contributions to algebra, number theory, calculus, differential calculus, calculus of variations, celestial mechanics, and statistics, respectively.

1.5.3 19th Century

There were important advances in abstract algebra throughout the 1800s. Vector spaces were first conceptualized by Hermann Grassmann, while William Rowan Hamilton focused on non-commutative algebra. The foundations of mathematical logic were established by George Boole, whose algebra is today known as Boolean algebra. It has many uses in computer technology.

Mathematicians made great strides in examining the limits of mathematics during the 19th century. The Abel-Ruffini theorem, established by two eminent mathematicians, Évariste Galois of France and Niels Henrik Abel of Norway, states that solving polynomial problems with degrees more than four using standard algebraic techniques is not feasible. But in the 19th century, mathematicians also went beyond geometry's confines of three dimensions by employing hypercomplex numbers and taking into account parameter space. By doing so, they were able to expand the realm of geometry beyond the traditional three dimensions. The discoveries made by these mathematicians during the 19th century laid the foundation for further advancements in mathematics and science.

Two mathematicians who made substantial contributions to the study of polynomial equations were Abel and Galois. Their work laid a strong basis for the advancement of group theory and other abstract algebraic disciplines. Group theory is a crucial tool for comprehending the underlying laws of the universe since physicists and other scientists use it extensively nowadays to investigate symmetry in a variety of systems and events.

The London Mathematical Society was founded in 1865, followed by the Société Mathématique de France in 1872, the Edinburgh Mathematical Society in 1883, the American Mathematical Society in 1888, and the Circolo Matematico di Palermo in 1884. These national mathematical societies were founded during the 19th century. The Quaternion Society, the first global special-interest organization, was established in 1899 after a debate around a vector.

1.5.4 20th Century

Over the course of the 20th century, mathematics emerged as a prominent field of study, with a growing number of Ph. Ds in the field being conferred each year and a wealth of employment prospects in both the academic and industrial sectors.

David Hilbert's 1900 address to the International Congress of Mathematicians, in which he offered a list of 23 open mathematical problems, serves as a prime example of this era and had a significant influence on the field's mathematical study for the entire 20th century. For the most part, 20th-century mathematicians focused on these challenges, which covered a wide range of mathematical topics. Ten of the twenty-three difficulties have been resolved thus far, seven have only been partially resolved, and two are yet unanswered. The remaining four can't really be classified as solved or unresolved since their formulations are too vague.

A lot of important mathematical hypotheses were finally proven true in the 20th century. They used a computer in 1976 to prove the controversial four-color theory. Their names are Wolfgang Haken and Kenneth Appel. Andrew Wiles solved Fermat's Last Theorem in 1995 with the help of work done in the past.

In mathematics, there have been great relationships that have covered a huge amount of information and involved a huge number of people. One example is the categorization of finite simple groups, which is also known as the "enormous theorem." Proof of the theorem was made public between 1955 and 2004. It included tens of thousands of pages, or about 500 journal pieces written by about 100 people. One of the most famous scientists of the 20th century was Srinivasa Aiyangar Ramanujan. He was an Indian man who had learned on his own and had a lively personality that stood out. It is thought that Ramanujan made about 3000 conjectures or proved theorems in different areas of mathematics.

1.6 Different Types of Numbers in Mathematics

The classification of numbers is a fundamental concept in mathematics and possesses great significance in our daily lives. Numbers are categorized into different types based on their properties and representation on the number line. Each type of number exhibits unique characteristics that distinguish it from others. A comprehensive understanding of these classifications is indispensable for solving mathematical problems and performing operations efficiently. To aid with understanding, the following explanation offers a summary of the many kinds of numbers, along with information on their characteristics and instances.

1.6.1 Natural Numbers

The term "counting numbers" refers to a collection of positive integers, from 1 to infinity, which are utilized for counting. These numbers are also called natural numbers. The set of natural numbers, which is commonly represented by the letter "N," is all positive integers between 1 and infinity that do not have any fractional or negative values.

$$\mathbf{N} = \{1, 2, 3, 4, 5, \dots\}$$

1.6.2 Whole Numbers

A collection of non-negative integers without any fractional or decimal components is called a whole number. They are called natural numbers with zero quite a bit. The set of whole numbers is represented by the letter "W". The complete set of numbers is defined as follows:

$$\boldsymbol{W} = \{0,1, 2, 3, 4, 5, \dots\}$$

1.6.3 Integers

An integer is the set of all whole numbers, including a negative set of natural numbers. It is a basic mathematical concept. This set, which consists of zero, positive, and negative integers, is thought to be infinite in both positive and negative directions. A basic component of many mathematical theories and applications, integers are widely used in addition, subtraction, multiplication, and division, among other mathematical operations. The letter "Z," which stands for the set of integers, signifies the following:

$$\boldsymbol{Z} = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$$

1.6.4 Rational Numbers

A number is deemed rational if it can be expressed as the ratio of two numbers, p/q, where p and q are integers and q does not equal zero. A fundamental concept in mathematics, rational numbers have numerous applications in the fields of science and engineering, among others. It's crucial to keep in mind that all integers are

rational numbers since they can all be expressed as p/1. The concept of rational numbers serves as the foundation for the study of real numbers, which is essential for advanced mathematical analysis.

The set of rational numbers is defined as follows and is represented by the letter "Q":

$$\mathbf{Q} = \left\{ \frac{p}{q} \mid p, q \in Z \ \wedge q \neq 0 \right\}$$

1.6.5 Irrational Numbers

Any number that cannot be represented as the ratio of two integers is considered irrational in mathematics. The character "Q^'" stands for these numerals. It is significant to remember that irrational numbers are endless, non-repeating decimals in contrast to rational numbers, which may be expressed as a fractions. As a result, they are essential to many mathematical ideas, such as trigonometry and calculus.

1.6.6 Real Numbers

Real numbers, which include both rational and irrational numbers, are a basic idea in mathematics. It's important to remember that a number is considered real if it can be thought of, excluding complex numbers. In other words, real numbers provide a broad and encompassing set that is necessary for the understanding and implementation of several mathematical ideas. "R" or "R" is the letter that represents these numerals.

1.6.7 Complex Numbers

The formula for representing complex numbers is a+bi, where "i =√ (-1)" indicates an imaginary number and "a" and "b" are real integers. It's crucial to remember that "a" represents the real component of a complex number and "b" represents the imaginary part. The letter "C" stands for these numerals. Many disciplines, including engineering, mathematics, and physics, frequently employ complex numbers. When tackling a variety of issues involving complex arithmetic, the capacity to represent integers with both real and imaginary components is crucial.

Examples: $4 + 4i, -2 + 3i, 1 + \sqrt{2}i$, etc.

1.7 Self-Assessment Questions

1. What were the major financial activities of prehistoric humans, and how did they affect their societies?

2. What types of connections did prehistoric humans have with their surroundings, and how did their behavior impact the environment?

3. What kinds of material are used by researchers to link together prehistoric societies and cultures?

4. What is Babylonian mathematics? Describe some advantages of Babylonian mathematics.

5. Which fundamental concepts and methods in mathematics did the Babylonians introduce and develop?

6. What is the key architecture of Egyptian mathematics? What is significant?

7. How did the mathematical contributions of Thales and Nicomachus influence the development of Greek mathematics and its impact on modern mathematical believe?

8. Discuss the mathematical ideas and advancements of Plato and Eudoxus in the ancient Greek period.

Unit–2

PROBLEM SOLVING TECHNIQUES

Written by: Dr. Rizwan Salim Badar

Reviewed by: Dr. Muhammad Asad Zaigham

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2.1 Introduction

In this unit, firstly we would be defining parts and whole and their relationship. In the context of this, examples from real life be discussed. Secondly, we would discuss units and rates and see their applications from real life scenarios. The questions related to the above-mentioned topics be given at the end.

2.2 Objectives

The Unit 2 will enable the students to:

- Recognize the connection between the parts and the whole at the conclusion of this lesson
- Realistic life situations including both wholes and pieces
- Realistic life examples using rates and units
- Analysis of Units Tool for addressing problems

The main part of the chapter is divided into three sections including;

- Parts and whole and their relations including real world examples
- Units and rates followed by discussion from real life
- Usage of ‘Parts and whole’ and ‘Units and Rates’ in problem solving

2.3 Parts & Whole and Their Relationship

In this section, we will define the concepts of ‘parts’ and whole and understanding the relationship between parts and wholes is fundamental in various fields, including philosophy, mathematics, and systems thinking. The concept refers to how individual components contribute to the entirety of a system or entity. Let's break down the introduction of parts and wholes:

2.3.1 Parts

- Parts are the individual elements or components that collectively make up a whole.
- These components can vary in size, complexity, and function depending on the system or entity they belong to.
- Each part may have its own characteristics, properties, and interactions within the system.

2.3.2 Whole

- The whole represents the entirety, or the sum total formed by the combination of its parts.
- It is the unified entity that emerges from the integration or organization of its constituent parts.
- The whole often exhibits properties or behaviors that are distinct from its individual components.

2.3.3 Relationship

- The relationship between parts and wholes is one of interdependence and emergence.
- Parts interact with each other and contribute to the functioning and behavior of the whole.
- The whole, in turn, influences the behavior and properties of its constituent parts.
- Emergent properties, characteristics, or behaviors may arise at the level of the whole, which cannot be fully explained by examining its individual parts in isolation.

Examples

- In a biological organism, organs, tissues, and cells are the parts that together form the whole organism.
- In a mechanical system, individual components like gears, motors, and shafts constitute the parts, while the entire machine represents the whole.
- In a societal context, individuals, families, communities, and institutions are the parts contributing to the whole society.
- In mathematics, sets, subsets, and elements illustrate the concept of parts and wholes in a more abstract sense.
- If you have a jar containing 100 marbles, and you take out 25 marbles, then 25%25% represents the part (the number of marbles taken out) compared to the whole (the total number of marbles in the jar).
- Suppose you have Rs 500 and you spend Rs 200. Then Rs 200 represents the part of the money spent, while Rs 50 represents the whole amount you had initially. So, in decimal form, you spent 0.4 or 40% of your initial amount.
- The area of a rectangle may be split into two sections: the width and the length. The rectangle's total area is made up of its length and breadth.
- Each data point in a data set represents a portion of the total, and the collection of data points as a whole represents the full data set.
- In an algebraic expression like 2x+3, 2x and 3 are parts of the whole expression. The whole expression represents a combination of these parts.

Parts and wholes are integral concepts in various aspects of life, from simple everyday experiences to complex systems. Here are some real-life scenarios illustrating the relationship between parts and wholes:

- **Human Body:**
  The human body consists of various parts such as organs (heart, lungs, liver), tissues, cells, and molecules. Each part has its own function, but together they form a cohesive whole that sustains life.
- **Ecosystems:**
  Ecosystems are composed of numerous components including plants, animals, microorganisms, soil, water, and climate. Each element plays a role in maintaining the balance and functioning of the ecosystem as a whole.
- **Automobile:**
  An automobile is made up of several parts like the engine, transmission, wheels, chassis, and electrical system. Each part contributes to the overall functionality and performance of the vehicle.

- **Business Organizations:**
  Businesses consist of different departments such as marketing, finance, operations, and human resources. Each department has its own responsibilities, but they work together to achieve the company's goals and objectives.

- **Food Chain:**
  In a food chain, organisms are classified into producers, consumers, and decomposers. Each organism occupies a specific position in the chain, contributing to the flow of energy and nutrients within the ecosystem.

- **Computer Systems:**
  Hardware elements such as the central processing unit (CPU), memory, storage, input/output, and software make up computers. Every part is essential to the overall operation of the computer system.

- **Literature:**
  Literary works such as novels, poems, or plays are composed of various elements like characters, plot, setting, theme, and style. These elements work together to create a coherent and meaningful story or message.

- **Musical Composition:**
  A musical composition consists of different parts including melody, harmony, rhythm, and dynamics. Each element contributes to the overall structure and emotional impact of the piece.

- **Sports Team:**
  A sports team is comprised of individual players, each with their own skills and positions. Together, they form a cohesive unit working towards a common goal of winning games and championships.

- **Environmental Systems:**
  Climate systems involve components such as the atmosphere, hydrosphere, lithosphere, and biosphere. Each part interacts with the others, influencing weather patterns, climate conditions, and the overall health of the planet.

These examples demonstrate how parts and wholes are fundamental concepts that can be observed across various domains, illustrating the interconnectedness and complexity of systems in the world around us.

2.3.4 Importance

- Understanding parts and wholes is crucial for analyzing complex systems and phenomena.

- It provides insights into how systems are structured, how they function, and how they evolve over time.

- Recognizing the interdependence between parts and wholes is essential for effective problem-solving, decision-making, and innovation in various domains.

In essence, the concept of parts and wholes offers a framework for comprehending the interconnectedness and emergent properties of systems, from the microscopic level of atoms to the macroscopic level of societies and ecosystems.

2.4 Units and Rates

Units and rates are fundamental concepts in mathematics and science, essential for understanding and quantifying various phenomena in the physical world. Let's delve into each concept in detail:

2.4.1 Units

Units are standardized quantities used for measurement. They provide a reference against which other measurements can be compared. Units can be classified into two main categories:

1. **Base Units:** All other units are generated from these fundamental units. For example, consider:
   - Meter (m) for length and kilogram (kg) for mass
   - The temporal second (s)
   - Kelvin (K) for temperature; Ampere (A) for electric current
   - Mandela (cd) for luminosity intensity
   - Mole (mol) for material quantity

2. **Derived Units:** These units are created by combining different basic units. For example, consider:

- Newton (N) as a force unit (kg/s^2).
- Joule (J): kilograms per second (kg/s).
- Pressure (N/m2) in Pascals (Pa)
- For frequency (1/s), Hertz (Hz)

Units allow us to express measurements in a standardized and universally understandable way. They are crucial for communicating precise information about quantities.

2.4.2 Rates

A rate is a comparison of two different quantities, often with different units, expressed as a ratio. Rates are used to describe how one quantity changes concerning another quantity. Rates are typically expressed in terms of a specified unit of time or another unit of measure. Here are some common types of rates:

1. **Speed/Velocity:** A measure of an object's speed is the distance traveled in a given amount of time, commonly represented in meters per second or miles per hour. Direction and speed are both components of velocity.

2. **Acceleration:** Usually expressed in meters per second squared (m/s2), acceleration is the rate at which velocity changes in relation to time.

3. **Rates of Growth/Decay:** Rates, such as the interest rate or population growth rate, are used to characterize changes in quantity over time in disciplines like biology and economics.

4. **Density:** Density is a mass per unit volume measurement that is commonly given in kg/m^3 or grams per cubic centimeter (g/cm^3).

5. **Frequency**: Frequency is the rate of occurrence of a repeating event per unit of time, commonly measured in Hertz (Hz) or cycles per second.

6. **Conversion Rates:** Conversion rates describe the relationship between two different units of measure, such as currency exchange rates or conversion between metric and imperial units.

Here are some examples of units and rates from real-life scenarios:

1. **Grocery Shopping:**
- **Units:** Purchasing items by weight (e.g., kilograms of fruits, grams of spices).
- **Rates:** Comparing prices per unit (e.g., price per kilogram of apples) to determine the best value for money.

2. **Driving:**
- **Units:** Measuring distances in kilometers or miles.
- **Rates:** Monitoring speed in kilometers per hour or miles per hour to adhere to speed limits and estimate travel time.

3. **Cooking:**
- **Units:** Using measurements such as cups, teaspoons, and ounces for ingredients.
- **Rates:** Following cooking instructions that specify rates of change, like simmering for a certain number of minutes.

4. **Exercise:**
- **Units:** Tracking distance covered in kilometers or miles during a run or walk.
- **Rates:** Monitoring heart rate in beats per minute to maintain a desired level of intensity during exercise.

5. **Banking:**
- **Units:** Managing finances in currency units (e.g., dollars, euros).
- **Rates:** Calculating interest rates on loans, mortgages, or savings accounts to understand financial implications.

6. **Fuel Consumption:**
- **Units:** Measuring fuel consumption in liters or gallons.
- **Rates:** Calculating fuel efficiency in kilometers per liter or miles per gallon to optimize driving habits and vehicle performance.

7. **Home Utilities:**
- **Units:** Tracking energy usage in kilowatt-hours (kWh) for electricity and cubic meters for water.
- **Rates:** Analyzing utility bills to monitor consumption rates and identify potential energy-saving opportunities.

8. **Internet Usage:**
- **Units:** Monitoring data usage in gigabytes (GB) or megabytes (MB).
- **Rates:** Understanding internet speed in megabits per second (Mbps) to ensure sufficient bandwidth for streaming, gaming, or working from home.

9. **Fitness Tracking:**
- **Units:** Recording steps taken in a day using a pedometer or fitness tracker.
- **Rates:** Setting goals for daily step counts or tracking progress over time to maintain an active lifestyle.

10. **Temperature Control:**
- **Units:** Setting room temperature in degrees Celsius or Fahrenheit.
- **Rates**: Adjusting heating or cooling systems to maintain a comfortable indoor environment.

11. **Travel Planning:**
- Units: Estimating travel time in hours or days for a trip.
- Rates: Researching transportation options based on rates of speed and cost to determine the most suitable mode of travel.

12. **Construction:**
- **Units:** Measuring materials such as lumber and concrete in feet, meters, or cubic meters.
- **Rates:** Calculating construction timelines based on rates of progress and productivity.

In these scenarios, units and rates play crucial roles in everyday decision-making, resource management, and goal setting, demonstrating their significance in practical applications across various aspects of life.

2.5 Self-Assessment Questions

The students are hereby given the exercise questions on “Parts and Whole” and “Units and Rates” for self-assessment of the topics given in the chapter as follows:

1. Eight equal parts are cut out of a circle. What percentage of the whole circle is one part?
2. There are 5 equal parts that make up a square. How much of the whole is dark if three parts are shaded?
3. If a pizza is cut into 12 equal slices and 8 slices are eaten, what fraction of the pizza remains?
4. Thirty of the fifty kids are boys and twenty are girls. How many of the people in the group are boys? How many of them are girls?
5. A jar contains 120 marbles, of which 3/5 are blue and the rest are red. How many red marbles are there?
6. Four equal sides of a triangle are cut in half. What percentage of the whole is one side?
7. John has 24 apples. He gives away 3/8 of them to his friends. How many apples does he give away?
8. Cut a 2-meter-long ribbon into five equal sections. What is each piece's length?
9. There are 80 balls in a bag; 25 of them are green and the remaining 45 are red. What is the number of green balls?
10. What percentage of the entire is one portion if a line segment is split into seven equal parts?
11. A tank contains 500 liters of water. If 3/10 of the water is removed, how many liters remain?
12. A school has 600 students. If 2/3 of them are enrolled in the math club, how many students are in the math club?
13. A rectangle is divided into 9 equal parts. If 5 parts are shaded, what fraction of the whole is shaded?
14. What percentage of a cake is left after ten evenly sliced portions are consumed?

15. A rope that is 15 meters long is cut into 3 equal pieces. What is the length of each piece?

16. In a group of 80 people, 3/8 are adults and the rest are children. How many adults are in the group?

17. A box contains 200 chocolates, of which 3/5 are milk chocolates and the rest are dark chocolates. How many dark chocolates are there?

18. If a square is divided into 16 equal parts, what fraction of the whole is one part?

19. Sarah has 36 pencils. She gives away 5/9 of them to her classmates. How many pencils does she give away?

20. A rope that is 18 meters long is cut into 6 equal pieces. What is the length of each piece?

21. If a line segment is divided into 9 equal parts, what fraction of the whole is one part?

22. A bag contains 150 marbles, of which 2/3 are blue and the rest are red. How many red marbles are there?

23. A rectangle is divided into 6 equal parts. If 4 parts are shaded, what fraction of the whole is shaded?

24. If a cake is cut into 8 equal slices and 2 slices are eaten, what fraction of the cake remains?

25. A pool contains 800 liters of water. If 5/8 of the water is removed, how many liters remain?

26. Convert 2.5 meters to centimeters.

27. If a car travels at a speed of 60 miles per hour, how many kilometers per hour is that?

28. If a train travels 300 kilometers in 5 hours, what is its speed in kilometers per hour?

29. Convert 500 grams to kilograms.

30. If a rectangle has sides of length 6 cm and 8 cm, what is its area in square centimeters?

31. If a tank can hold 1000 liters of water, how many cubic meters of water can it hold?

32. A car travels 240 miles in 4 hours. What is its speed in miles per hour?

33. Convert 3.5 hours to minutes.

34. If a bicycle travels 15 kilometers in 45 minutes, what is its speed in kilometers per hour?

35. If a person runs at a speed of 10 meters per second, how many kilometers per hour is that?

36. Convert 150 centimeters to meters.

37. If a train travels at a speed of 80 kilometers per hour, how far will it travel in 3 hours?

38. A rectangular prism has dimensions of 4 meters by 3 meters by 2 meters. What is its volume in cubic meters?

39. Convert 500 milliliters to liters.

40. If a car travels at a speed of 50 miles per hour, how many meters per second is that?

41. If a rectangle has an area of 48 square meters and a length of 8 meters, what is its width?

42. A plane travels 1500 kilometers in 2 hours. What is its speed in kilometers per hour?

43. Convert 2.5 kilometers to meters.

44. If a train travels 480 miles in 6 hours, what is its speed in miles per hour?

45. If a tank can hold 5000 liters of oil, how many cubic meters of oil can it hold?

46. A car travels at a speed of 72 kilometers per hour. How far will it travel in 2.5 hours?

47. Convert 2.75 meters to millimeters.

48. If a person walks at a speed of 5 kilometers per hour, how many meters per minute is that?

49. A swimming pool measures 25 meters in length, 10 meters in width, and 2 meters in depth. What is its volume in cubic meters?

50. If a train travels at a speed of 100 miles per hour, how far will it travel in 2.5 hours?

Unit–3

NUMBERS AND UNIVERSE

Written by: Dr. Bismah Jamil

Reviewed by: Dr. Muhammad Asad Zaigham

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3.1 Introduction

In this Unit, we will explore the relationships between numbers, the building blocks of mathematics and the cosmic universe. We shall set out on an exploration to understand the complex relationship between the world of numbers and the mysteries of the universe, from reading the mathematical laws variable the universe to uncovering its difficulties. Numbers were one of the first structured mathematical objects known to mankind. Although numbers formally exist now, their essence and use were well known before. The oldest kind of numbers known to humans were natural numbers. These numbers were also called counting numbers. As the name suggests the numbers were primarily used for counting purposes. After that, the set integers were discovered. The discovery of integers leads to rational and irrational numbers and eventually the set of real numbers.

3.2 Objectives

After completing this unit, Students will be able to learn about:

- Understanding set numbers and their types
- Rational and irrational correlations
- Big numbers and small numbers
- Understanding Mathematical Uncertainty
- Applications of Mathematical Uncertainty
- Finance and risk management
- Artificial intelligence and machine learning.

3.3 History of Numbers and Number System

The concept of numbers and number systems is an ancient concept known to humans. Some known number systems are listed below:

3.3.1 Early Counting Systems

Counting objects is thought to be the first of the mathematical problems that humans face. The use of counting was utilized by cattle keepers to count the inflow and outflow of their animals. Humans have been using various methods to count objects such as tally marks, stones, or strike marks on surfaces like sand or on the walls of caves. These were regarded as one of the early counting systems and were based on simple labelling of objects.

3.3.2 Babylonian Numerals

The Babylonian numeral system is one of the oldest numeral systems known. Its origin dates to around 3000 BCE. It was a positional system based on the number 60 and used a combination of various symbols to represent different numbers.

Example: For the number 20, the Babylonian numeral was(□□).

3.3.3 Egyptian Numerals

The ancient Egyptians also developed their numerals around 3000 BCE. They utilized the symbols to represent powers of 10 and used special symbols for fractions.

Example: 87 is represented as four coils of rope (□□□□) followed by a hobble for cattle (□) and two single strokes (□□).

3.3.4 Roman Numerals

Another numeral system known as the Roman numeral system was developed around 500 BCE. It was widely used in the Roman Empire and neighbouring regions. It used combinations of Latin letters to assign different numeric values. While it was useful for practical purposes like counting and record-keeping, it lacked the efficiency of later positional systems.

3.3.5 Decimal System

The decimal system was regarded as one breakthrough in the study of number systems. It evolved around the 5th century CE. This system was later transmitted and used in the West through the Arabic world, revolutionized mathematics and is regarded as the foundation for modern arithmetic.

Example: The number one hundred and twenty is represented by 120.

3.3.6 Arabic Numerals

The decimal system was further refined by various scholars in the Islamic Golden Age. It spread throughout the Islamic world and eventually reached Europe, where it replaced Roman numerals for being more practical and easier to use. The digits ranged from 0-9 and the positional notation system made arithmetic much more efficient.

3.3.7 Modern Number Systems

Today, we mainly use a combination of Decimal-Arabic numeral system, a positional decimal system with digits ranging from 0 through 9. This system forms the basis for all mathematical operations and is used universally in science, engineering, commerce, and everyday life.

3.4 Some Known Sets of Numbers

In mathematics, there are many types of numbers, each number has its own characteristics and properties. Let us quickly overview of some common types of numbers:

3.4.1 Natural Numbers $\mathbb{N}$

Natural numbers are known as counting numbers. These numbers start from 1 and extend infinitely. These numbers do not include zero or negative numbers. They are denoted by the symbol $\mathbb{N}$. The set is given by {1, 2, 3, 4, ...}

3.4.2 Whole Numbers $\mathbb{Z}^+$

The whole numbers are the natural numbers together with zero as well like natural numbers, these numbers also extend to infinity. They are denoted by symbol $\mathbb{Z}^+$. The set is given by {0, 1, 2, 3, ...}.

3.4.3 Integers $\mathbb{Z}$

The set of Integers includes all positive and negative natural numbers, as well as zero. They extend infinitely in both directions. They are denoted by symbol $\mathbb{Z}$. The set of integers is given by {..., -3, -2, -1, 0, 1, 2, 3, ...}.

3.4.4 Rational Numbers $\mathbb{Q}$

The set of rational numbers are numbers that can be expressed as $\frac{p}{q}$ where $p, q \in \mathbb{N}$. These numbers can be finite as well as recurring decimals. They include integers and fractions as well. They are denoted by the symbol $\mathbb{Q}$**.** Some examples of rational numbers are 1/2, 3/4, -5/6, 2, -3, 0.5, 1.333...

3.4.5 Irrational Numbers $\mathbb{Q}'$

One might think of irrational numbers as the set of rational numbers' counterparts. This indicates that there is no way to describe these values as a ratio of two natural integers. The decimal representations of these integers are non-repeating and non-terminating. The sign Q' is used to represent them. e (Euler's number), π (pi), and the square root of 2 (√2) are a few instances of irrational numbers.

3.4.6 Real Numbers $\mathbb{R}$

Both rational and irrational numbers are included in the set of Real numbers. They belong to the biggest number class. They consist of irrational numbers, fractions, and integers. The letter R is used to represent them. Real numbers include, for instance, -√2, -1/3, 0, 3.14, √5, ...

3.4.7 Complex Numbers $\mathbb{C}$

The collection of complex numbers consists of integers of the form a + bi, where i = √-1 and a and b are real numbers. The real number system is extended by complex numbers to allow solutions to equations without real roots. The letter C is used to represent them. These numbers include, for instance, 2 + 3i, -1 - 2i, 4i,...

3.5 The Numbers and the World Around US

Understanding the world through numbers is one of the fundamental aspects of various disciplines, including natural science, economics, sociology, and many other disciplines. Let us go through some concepts that help us to understand the world using numbers.

3.5.1 Quantification and Measurement

Numbers allow us to quantify and measure various objects and aspects of the world. These things range from physical quantities like distance, mass, and time to abstract concepts like happiness, economic growth, and social inequality. All these things are quantified by using numbers helping us evaluate their strengths or weaknesses.

3.5.2 Data Analysis

Numbers are essential for collecting, organizing, and analyzing data. Usually, data is recorded in the form of numbers. By making use of statistical methods, data analysts can identify patterns, trends, correlations, and relationships in data sets, which can provide insights into complex phenomena. This helps key decision-makers to have better insight into the data they are working on.

3.5.3 Prediction and Modelling

Once data is analyzed. The patterns are discovered in the data. This helps in the development of mathematical models to predict future outcomes or understand the behavior of systems. One such application can be seen in the areas of weather forecasting, financial markets, and epidemiology.

3.5.4 Decision Making

The Numbers obtained after critical data analysis play a significant role in decision-making processes. This helps decision makers to make decisions based on data. It helps in optimized allocation of resources, setting policies, or evaluating risks. This yields better operational output for an organization.

3.5.5 Communication and Visualization

Numbers in data often appear to be random at first glance. However, the use of Graphs, charts, and visualizations helps us convey information visually, making it easier to understand and interpret. This helps the data to be readily understandable to a large class of audience.

3.5.6 Comparative Analysis

The numbers allow us to effectively compare different aspects of the world. Comparative analysis is one of those areas, numbers help us compare two objects quantitively. Whether it's comparing the performance of different economies, analyzing demographic trends across countries, or evaluating the effectiveness of policies and interventions.

3.5.7 Verification and Falsification

Numbers also provide a basis for verifying hypotheses or denying a myth. Scientific research relies on numerical data that support or oppose claims, helping to advance our understanding of the world.

Example: Imagine a city named A that has 10,000 residents at the beginning of 2024. There are 500 documented deaths annually compared to 700 typical annual births. It is easy to use basic mathematics to estimate the population by 2026. The net population at the end of 2025 would be 10,000 - 500 + 700 = 10,200. The net population at the end of 2026 would be 10,200 - 500 + 700 = 10,400. For the administrator of City A's resource planning, this number is quite useful. Therefore, even seemingly simple numbers with predetermined meanings might provide an alternative viewpoint on reality.

3.6 Big and Small Numbers

Big and small numbers are two extremes in numbers and have several applications in various fields, and their significance varies depending on the context. Here's a more detailed exploration of their applications. We begin with the application of some big numbers:

3.6.1 Big Numbers

Economics and Finance:

i) **National GDP:** Big numbers are used to represent the economic output of a country, such as the GDP of the country. The GDP of a country is a big number, e.g. United states GDP is in the trillions of dollars.

ii) **Stock Market:** Each day stocks of business companies trade in the stock market. The capital investment of stock is in billions of rupees. The market capitalization of large corporations can reach hundreds of billions or even trillions of dollars.

Astronomy:

i) **Astronomical Distances:** In our galaxy, astronomical objects are way apart. Therefore, distances between celestial objects are often vast, measured in light-years. For example, the distance to the nearest known star, Proxima Centauri, is about 4.24 light-years.

ii) **Stellar Mass:** The large mass such as of stars known as stellar mass can range from a few times the mass of the Sun to millions of times the Sun's mass. This is yet another example of big numbers.

Population and Demographics:

i) **Global Population:** Global Population is a big number which changes with each passing time. The current global population is estimated over 7 billion people, representing a big number in demographics. In megacities like Tokyo or Karachi populations in the tens of millions.

***Scientific Notation*:**

i) **Avogadro's Number:** This number is used in chemistry, Avogadro's number (6.022 × 10^23) represents the number of atoms or molecules in one mole of a substance. This is another example of a big number.

ii) **Planck's Constant:** In physics, Planck's constant (6.62607015 × 10^ (-34) m^2 kg / s) is a fundamental constant that is used to describe the behavior of particles and waves at the quantum level.

3.6.2 Small Numbers

Now we turn our attention to small numbers. A small number is classified to be a number that is very small and is near zero. Like big numbers, small numbers also have applications. Some of them are listed below:

***Microbiology and Nanotechnology*:**

i) **Cell Size:** The size of single cells or a microorganism is very small and is often measured in micrometers (μm) or nanometers (nm), such as the size of bacteria or viruses.

ii) **Nanoparticles:** A nanoparticle is a particle of (1nm=10^-9m). In nanotechnology, a particle with dimensions on the order of nanometers is manipulated for various applications, such as drug delivery systems or electronics. These nanoparticles are also an additive of antifreeze agents in automobiles.

Mathematics and Statistics:

i) **Probability:** Small numbers in probability represent the occurrence of rare events, such as the likelihood of winning the prize with 1 million participants or the chance of a specific genetic mutation occurring.

***Chemistry*:**

i) **Atomic Sizes:** The radius of atoms and their bond lengths are often very small in the order of angstroms (10^ (-10) meters), representing very small distances.

ii) **Molecular Weights:** As the size molecular weights of various compounds are often measured in small numbers of Daltons (Da), which are small units of mass used for the measurement of molecules.

***Finance*:**

i) **Fractional Currency:** In financial transactions, fractions of a currency unit, such as cents or pence, are commonly used for pricing goods and services. Also, small numbers come into play when a weaker currency is converted to big currency. For example, converting Colombian pesos to US dollars.

Example: Consider the measurement of a liquid, usually it is done in milliliters or liters. Consider liter1 liter this is a small number but once we convert it to ml it is 1000ml which is a large number. Therefore, in measurement, the same quantity using two different units can be small and large numbers.

3.7 Mathematical Uncertainty

3.7.1 Introduction

Mathematical uncertainty arises naturally because of limitations of our knowledge and the models used in describing real-world phenomena with absolute certainty. Even the smallest uncertainty present in a mathematical model of various disciplines, from physics to finance, brings challenges and opportunities for decision-making and problem-solving. Now we formally understand mathematical uncertainty.

3.7.2 Understanding Mathematical Uncertainty

Mathematical uncertainty is born from several sources, this ranges from measurement errors, incomplete information, and built-in randomness in the systems. It makes the process of precise predictions or determinations about future outcomes a challenging task. Probability theory provides a complete framework for quantifying uncertainty, allowing us to assess the occurrence of different events based on available evidence and assumptions.

3.8 Applications and Implications in Various Fields

3.8.1 Finance and Risk Management

Financial markets are inherently uncertain, and influenced by various factors such as economic indicators, investor behavior, and geopolitical events.

Quantitative finance utilizes mathematical models, such as stochastic calculus and option pricing theory, to assess and manage financial risk.

Monte Carlo simulation techniques help investors and businesses analyze the potential outcomes of investment decisions under uncertain market conditions.

3.8.2 Engineering and Design

Engineers encounter uncertainty when designing structures, systems, and products, as they must account for variability in material properties, environmental conditions, and operational parameters.

Probabilistic methods, such as reliability analysis and sensitivity analysis, aid in designing robust and resilient engineering systems capable of withstanding uncertain conditions and unforeseen events.

3.8.3 Climate Science and Environmental Management

Climate models incorporate mathematical uncertainty to simulate future climate scenarios and assess the potential impacts of climate change.

Uncertainty quantification techniques help policymakers evaluate the effectiveness of mitigation and adaptation strategies for addressing environmental challenges, such as sea-level rise and extreme weather events.

3.8.4 Healthcare and Medical Decision Making

Medical diagnosis and treatment decisions involve uncertainty due to variability in patient responses, diagnostic accuracy, and treatment outcomes.

Bayesian statistics and decision analysis assist healthcare professionals in evaluating diagnostic tests, predicting patient outcomes, and optimizing treatment strategies based on available evidence and patient preferences.

3.8.5 Artificial Intelligence and Machine Learning

Machine learning algorithms encounter uncertainty when dealing with noisy or incomplete data, model complexity, and inherent uncertainty in real-world phenomena.

Probabilistic machine learning approaches, including Bayesian inference and probabilistic graphical models, enable algorithms to make robust predictions and quantify uncertainty in their outputs.

3.8.6 Implications of Mathematical Uncertainty

Embracing mathematical uncertainty has several implications for decision-making, policy formulation, and societal resilience:

Decision-makers must acknowledge and communicate the inherent uncertainty in their predictions and recommendations to avoid overconfidence or unwarranted certainty.

Robust decision-making frameworks that account for uncertainty can lead to more resilient and adaptive strategies in various domains, from infrastructure planning to public health management.

Enhancing public understanding of mathematical uncertainty fosters critical thinking, informed decision-making, and a more scientifically literate society capable of grappling with complex issues such as climate change and pandemics.

Examples: if it is not known whether or not it will rain by tomorrow, then there is a state of uncertainty. If we apply probabilities to find the possible outcomes using weather forecasts, the uncertainty has been quantified. Suppose we have quantified that as a 90% chance of sunshine. If there is a soccer match planned for tomorrow anticipating a huge revenue in terms of tickets then there is a risk since there is a 10% chance of rain, and this chance of rain is undesirable. Furthermore, it is a business event and involving 100,000 USD would be lost if it rains, then the risk has been quantified (a 10% chance that a loss of 100,000 USD is adhered).

3.9 Self-Assessment Questions

1. Classify the following numbers into their number sets;

   a. 2

   b. $\sqrt{3}$

   c. $\frac{5}{3}$

2. Give examples of big and small numbers in a fixed scenario.

3. Using the example of population death rate and birth rate. Find in how many years the population has increased to 11000.

4. A person invests 10,000 PKR in a newly constructed water park ride. The person gets profit when the ride is taken by park visitors. It is estimated that there is a 10% chance that no one will visit the water park. Find out the likelihood that the investment is successful.

5. Consider a five-sided dice having zeros on all four faces and a hundred on the fifth face. What is the chance that if the dice are rolled the person gets the number 100?

6. Consider two candidates who appeared for a selection of the post of senior scientist. Both persons were thoroughly examined by a panel of examiners. Person A got 80 points out of 100 while person B got 70 points out of 100. Scientist A got the position. Explain how numbers helped the panel of examiners to make the decision for selection.

7. Three companies of toothpaste competed in an open market. They produced a unit of toothpaste having the same retail price of 150PKR. Company A, B, and C sold 1200, 1100 and 1000 units per year respectively. The profit margins for companies A, B, C were 10PKR, 30PKR, and 5PKR respectively for each unit. Using comparative analysis point out the company having the highest profit.

8. A weighing scale has an uncertainty of maximum+5 grams per 1 kg. A load of 105kg is weighed on this scale. Find what could be a possible error that the scale can give when this load is weighed.

Unit–4

FINANCIAL ISSUES

Written by: Dr. Muhammad Nazam

Reviewed by: Dr. Muhammad Asad Zaigham

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4.1 Introduction

This unit covers money management concepts such as profit, loss, discount, zakat, simple interest, compound interest, and taxation. It also covers money management in real-world contexts such as federal budgeting and investments. To help solidify the principles, activities and examples are provided.

4.2 Objectives

After the successful learning of this unit, students will be able to:

- Understand and practice concepts of profit, loss, discount, zakat, simple interest, compound interest and taxation in their daily lives
- Understand the national and domestic budgeting
- Apply the concepts of rate, ratio, percentage, and fraction in their social activities

4.3 Money Management

This part will teach us how to calculate taxation, simple interest, compound interest, zakat, profit, loss, and percentage. Everyone must cope with these phrases because they are quite prevalent. We pay attention to the phrases%, profit, loss, and discount when we go shopping at the market. In Islamic finance, the term "zakat" denotes an individual's obligation to donate a certain portion of their yearly wealth to charity organizations. For Muslims, zakat is an obligatory ritual that is considered a kind of prayer. The worth of property and income are the basis for zakat. For those who meet the requirements, the typical minimum amount is 2.5%, or 1/40 of a Muslim's overall wealth and savings.

In banking, the terms for interest are simple interest and compound interest. The balance, or starting amount, of a loan is used to figure out simple interest. You can think of compound interest as "interest on interest" because it is determined by adding up both the principal amount and the interest that has been earned over time.

4.3.1 Taxation

The act of levying or imposing a tax by a taxing body is referred to as taxation. Income, capital gains, and estate taxes are among them.

Different Types of Taxation: These can include (but are not limited to):

- **Income tax:** People and businesses alike that are subject to governmental jurisdiction are liable for income taxes.
- **Corporate tax:** This type of tax is assessed against the earnings of a business.
- **Capital gains:** Capital gains taxes apply to any profits or gains that people or businesses make from the sale of certain assets, such as stocks, bonds, or real estate.
- **Property tax:** The local government levies property taxes, which are the owner's duty. This tax is calculated using the valuations of the properties and land.
- **Inheritance:** A type of tax levied on beneficiaries of another person's estate.
- **Sales tax:** A consumption tax that the government imposes on goods and services. Examples of this include excise taxes, state or municipal sales taxes, goods, and services taxes (GST), and value-added taxes (VAT).

In the following, we learn how to compute the above-mentioned terms.

4.3.2 Profit

It's a term used to describe financial gain, particularly the difference between earnings and expenses incurred in purchasing, running, or creating an object. It is computed in this way:

$$\text{Profit} = \text{Sale Price} - \text{Cost Price}$$

or

$$\text{Profit} = \text{Total Revenue} - \text{Total Expenses}$$

Example: An item is purchased for 675 rupees and sold for 900 rupees. Find the profit.

Explanation: Here Sale Price (S. P.) =900, and Cost Price (C. P.) =675, so that

Profit = 900-675=225.

Now if we are asked to find the profit (gain) percentage, we can find it by using the following formula.

$$\text{Profit } \% \ = \ \frac{\text{Profit}}{\text{C. P.}} \times 100\%$$

In Example 1, $\text{Profit } \% \ = \ \frac{225}{675} \times 100\% = \frac{331}{3}\%$.

Example: At a profit of Rs. 15 per meter of cloth, a seller sells 85 m of cloth for Rs. 8,925.

How much does one meter of cloth cost?

Explanation: The Trader sells 85 m of cloth for Rs. 8,925, so the sale price of 1m cloth is $\frac{8925}{85}$ = 105. We are given that the profit on 1m cloth is Rs. 15. Thus, C. P.= S. P. $-$ Profit = 105-15=90.

Example: When a product is sold for Rs. 800, a shopkeeper gains twenty-five percent profit. How much should he charge for the article in order to lose 25% of it?

Explanation: We are given that S. P. is 800 and profit % is 25%. To calculate the required S. P., we need to find C. P. Let x be the C. P., then, profit $= 800 - x$. Thus,

$$25\% = \frac{\text{profit}}{\text{C. P.}} \times 100\% = \frac{800 - x}{x} \times 100\%.$$

$$\frac{25}{100} = \frac{800 - x}{x} \text{ implies that } 25x = 80000 - 100x \text{ or } 125x = 80000.$$

Thus, $x = \text{C.P.} = 640$. Now, we need S. P. subject to a loss of 25%. Now the amount of loss is given by

$$\text{Loss} = 25\% \times 640 = 160.$$

Hence, S. P. = $640 - 160 = 480$ (Remember that Loss = C. P. – S. P).

4.3.3 Discount

The amount or percentage subtracted from an item's regular selling price is referred to as the discount. A discount denotes a drop in the cost of an item or service. The following formula is used to compute it:

$$\text{Discount} = \text{Marked price} - \text{Sale price}.$$

If the discount is given as a percentage, then it is calculated by the formula:

$$\text{Amount of discount} = \text{Marked price} \times \text{rate of discount}.$$

Example: The standard cost of a jacket was fifty rupees. There was a 10% discount written on it. Once the discount is applied, find the jacket's pricing.

Explanation: The marked price is Rs.50, and the discount rate is 10%. This means that the discount amount is

$$\text{Amount of discount} = 50 \times 10\% = 50 \times \frac{10}{100} = Rs.5.$$

$$\text{The price of the jacket} = Rs.50 - Rs.5 = Rs.45.$$

To find the discount percent we have the following formula:

$$\text{Discount } \% = \frac{\text{Discount}}{\text{Marked Price}} \times 100\%$$

If the discount percentage is given, we can find the Marked price and Sale price as follows:

$$\text{Marked price} = \frac{100 - \text{Discount}\%}{100} \times \text{Sale price}$$

$$\text{Sale price} = \frac{100}{100 - \text{Discount}\%} \times \text{Marked price}$$

Example: In some grocery stores, Rs. 12 cases of soda are being sold at a 20% discount. What will be the discount? What will be the sale price of the case of soda?

Explanation: Here the marked price is Rs. 12 and the discount rate is 20%. The amount of discount is given by

Amount of discount = $12 \times 20\%$= Rs. 2.40.

The sale price $= \ 12 - 2.40 \ = \ 9.6$.

4.3.4 Zakat

It is an amount paid annually by Muslims at the rate of 2.5% of the total amount saved till the first Ramadan of every Islamic year. It is obligatory for Muslims to have a prescribed amount (called "Nisab=7.5 tola of gold*"*). Zakat is a wealth-related kind of worship. It ensures that money is distributed fairly throughout society and keeps it from building up in the hands of a select few. It promotes equality, fraternity, and kindness in the community. Zakat is a useful tool for keeping wealth in circulation since it encourages investment and deters hoarding in the community. If people do not invest their riches, it will eventually be eaten by zakat. ALLAH says:

"Spending on zakat is restricted to the underprivileged, the needy, those hired to collect it, mending hearts [for Islam], releasing prisoners (or slaves), those in debt, the sake of Allah, and the [stranded] traveler-an obligation [mandated] by Allah. Allah is a Knowing and Wise Being." **(Al-Tawba: 60).**

Let Mr. A have Rs. x, then Zakat to be paid =2.5 % of $x = \frac{x}{40}$. If Mr. B has paid Rs. 250000 Zakat, then the total amount Mr. B has is =Rs. $25000 \times 40 =$ Rs. $10{,}000{,}000$.

4.3.5 Simple Interest and Compound Interest

There are two ways to calculate interest: simple interest and compound interest. Simple interest is calculated based on the principal, or the whole amount borrowed. Since compound interest is computed using both the principal and the interest that has accrued over previous periods, it is frequently referred to as interest on interest. Let's make use of these symbols:

- t_0: the lending instant, or the current time point.
- T or n: the loan's duration.
- P_0: the amount lent at that moment t = 0.
- S_T: the total amount of dischargeable debt at time t=T.
- I stand for interest and interest money.
- d: for the discount rate, and
- i: for the interest rate (of accumulation).

The absolute value of the return from lending the money in any form is the interest money, or interest $I = (S_T - P_0)$. Here's how simple interest is calculated:

$$I = P_0 \times i \times T.$$

For instance, let's imagine a student taking out a simple-interest loan with an annual interest rate of 6% to cover the $18,000 cost of college tuition for a single year. Over a three-year period, the student repays the debt. The sum of money paid in simple interest is:

$$\mathrm{I} = \mathrm{Rs.}\,3{,}240 = \mathrm{Rs.}\,18{,}000 \times 6\% \times 3.$$

On the other hand, compound interest (CI) is interest on interest that accrues and is added to the interest that has accumulated over earlier periods. The compound interest formula is:

$$CI = P_0 \times (1 + i)^t - P_0.$$

Here are a few instances of both compound and simple interest.

Example: Let's say you deposit Rs. 5,000 into a certificate of deposit (CD) for a year that yields 3% annual simple interest. After a year, the interest you would receive would be:

$$I = \mathrm{Rs.}\,5000 \times 3\% \times 1 = Rs.\,150.$$

Let's say that you can cash your certificate of deposit whenever you choose, and you will receive interest based on a prorated amount. What is the interest rate that you would receive if you cashed the CD after four months? You would be given:

$$Rs.\,50 = Rs.\,5{,}000 \times 3\% \times \frac{4}{12}.$$

Example: Assume Bob gets a three-year loan of Rs. 500,000 from his wealthy uncle, who also promises to charge Bob simple interest at a rate of 5% each year. What would Bob's annual interest costs be in addition to the total amount of interest he would pay after three years?

$$\mathrm{Rs.}\,500{,}000 \times 5\% \times 1 = 25000,$$

or a total interest charge of Rs. 75,000 after three years.

Example: In keeping with the previous scenario, Bob will require a three-year loan for an extra Rs. 500,000. Sadly, his wealthy uncle has lost all of his money. Thus, he takes out a bank loan with a five percent yearly interest rate that is compounded annually; the loan balance and interest are due in full after three years. What would Bob have to pay in interest overall?

Since compound interest is calculated on the principal and accumulated interest, here's how it adds up:

After Year One, Interest Payable

$$= \text{Rs. } 500{,}000 \text{ (Loan Principal)} \times 5\% \times 1 = 25000.$$

After Year Two, Interest Payable

$$= \text{Rs. } 525{,}000 \text{ (Loan Principal + Year One Interest)} \times 5\% \times 1 = 26250.$$

After Year Three, Interest Payable

$$= \text{Rs. } 551{,}250 \text{ (Loan Principal + Interest for Years One and Two)} \times 5\% \times 1 = \text{Rs. } 27562.$$

Total Interest Payable After Three Years

$$= \text{Rs. } 25{,}000 + \text{Rs. } 26{,}250 + \text{Rs. } 27{,}562.50 = 78{,}812.50.$$

It can also be calculated using the previously mentioned compound interest formula:
After three years, the entire interest payable is

$$= \text{Rs. } 500{,}000 \text{ (Loan Principal)} \times (1 + 0.05)^3 - \text{Rs. } 500{,}000 = 78{,}812.50.$$

This example demonstrates how paying interest on interest in addition to principal results in the compound interest formula.

4.4 Budget

Budgeting is a very important annual activity for almost all the countries. Every Government, either federal or provincial, makes a budget. It shows the actual figures of income and expenditure. It is prepared annually in advance. A budget period or financial year starts from 1st July and ends on 30 June every year. Every company and many families prepare their monthly and annual budgets. The following points are important in the preparation of the budget.

4.4.1 The Needs and Wants

Everybody has desires and needs. Our needs are the necessities for survival and development. We require clothing, food, drink, shelter, and health care. We desire great cars, movies, security, and a top-notch education. A lot of individuals mix up what they need and want. Since money is needed to satisfy both, which is more crucial? Yes, taking care of your needs has to come first! You won't have enough to meet your necessities if you prioritize gratifying your wants. It's crucial that you consider your options PRIOR to making any purchases. Our needs are never greater than our wants. We may become confused and have to consider a purchase for some time. Together, let's identify our requirements and wants. Food might be necessary.

4.4.2 What is Budget?

A budget is simply a plan for spending and saving your money- a smart tool you can use to manage your money for today and the future. It allows you to plan for the financial aspects of your life. The advantages of managing your money are that it can help you plan for purchases and expenses, and it is a way to track your spending, so you meet your financial goals.

Budgeting does not entail making rigid plans or adhering to regulations. Denying oneself enjoyable activities. However, it does entail handling your money with awareness and intelligence. Creating a strategy that suits your needs. In this manner, you get these benefits: aid in the control of spending and costs. aid in debt reduction or elimination. A strategy to grow your assets in the future.

We are going to learn about budgeting by talking about a sample budget process.

Consider the budget. A budget is a plan for how you're going to spend your money. If you're used to spending money without thinking, a budget is probably a new approach. It may also mean changing your definition of what a budget is. With a budget, you list the categories that you spend money on food, clothing, etc. Then, you set aside a certain amount of money for each category of expense. For example, if your rent is Rs.650 a month, you allocate that amount to housing. You then track your spending to make sure you don't spend more than the amount you budget. Over the course of the month, you write down everything you spend in each category. Then you'll know if you're going over the amount you allocated and can cut back, or maybe you need to adjust your budget. Find opportunities to eliminate expenses. Include irregular expenses (such as insurance). Keep your budget flexible. It is going to change, just as your income, expenses, goals, and needs change. Let's make our own budget.

So how do you create a budget? Base it on your history of spending. You start with how much you've spent on that category in the past, or with the fixed costs you know you have, like your rent. Use past receipts, credit card bills, and your checkbook register. You can analyze your past receipts and credit card bills to see how much you usually spend. Start tracking your spending now for a month or two. Or, if you haven't kept records, you can start now, by saving your receipts for the next few months. Also, include any expenses you may have on a quarterly or yearly basis as well. Be sure to include all incidental expenses, like newspapers, coffee, donations, etc. ASK: What are some irregular expenses you can think of? One of the things that influence how you budget your money for each category is whether the expense is fixed, non-fixed, essential, or desirable. Let's look at what I mean by these terms.

4.4.3 What is Income?

Money that comes to you through a job, social security, tips, or other benefits is considered income. Income can include salary, tips, other cash earned, bonuses, benefits/payments received, child support, alimony, etc. Your total earnings are

called GROSS income. On most income, you will have to pay taxes. The amount left after taxes is called NET income. Use your net income when figuring out your budget. Remember, that all income is not regular. If you work a part time job, your hours could increase or decrease. Or your employer may decide to reduce the number of employees and you could lose your job. Or you could get sick and not work. Income must be tracked and adjusted in your budget just like expenses. If you are divorced or receive support from another person, recognize that these payments could stop or be delayed. Emergencies will always come up so it's important to have money in savings to help you through tough times. One expense item in your budget should be emergency savings. (Figure out Marie's income or figure out your own using a blank sheet in the student guide)

4.4.4 What is Expense?

When you spend money, you have an expense. Expenses fall into categories such as food, entertainment, transportation, clothing, etc. It is important to list all areas in which you spend money – everything from big credit card purchases to cash spent on smaller items like morning coffee or a newspaper. Some expenses arise every month and may not change. These are called fixed expenses. Rent or a car payment are fixed expenses. Other expenses are flexible - they change each month. Grocery purchases, school supplies, and utilities may vary from month to month. Other expenses occur only a few times a year. Insurance payments or school tuition may come up every 3 or 4 months. We still must plan for them to be sure we have enough money to pay these bills.

4.4.5 Savings

Pay yourself before you spend any money. This indicates that you are saving money in order to accomplish your objectives. Decide how much you can deposit each time. Then follow the plan. Prioritize oneself on a regular basis. When you get paid, do every week or every month. Paychecks can be directly deposited into bank accounts. Then, a predetermined sum can be transferred to savings automatically by your bank. If you don't see the money, you won't miss it. Consider saving as an ongoing cost that, like rent or utilities, you MUST pay. Find out how much money is left over from paying yourself so that you can pay your bills.

Do not forget your emergency fund. You will still need to pay your expenses if you lose your job or get sick and can't work. You might need to rely on savings for short-term emergencies. Start building an emergency fund so it eventually has from 3 to 6 months' worth of your expenses. That might seem like a lot of money, but you don't have to create it all at once. You can start building it now and make regular contributions over time. Your emergency fund should be a savings priority.

Maintaining a lower level of expenses than revenue is crucial. Over time, debt accumulation becomes a major issue. Here are some tips to help you reduce your spending. Prioritize the things that you and your family truly need. A few essential

necessities are clothing, food, shelter, a way to get to work, and education. First, your needs have to be satisfied. Have a conversation with your family about what is actually required.

For example, you need food to eat. But you do not have to eat lunch in a restaurant. By making a sandwich at home, you could save money. Reduce the amount of what you want. Our wants will always outweigh our financial means. Consider WHY you want something when you discover it. How will this aid in my goal-achieving? Consider your reasons before making a new phone purchase. Is it a result of your friend's recent purchase? Are you attempting to win someone over? Do you really need a new phone? What can you save if you do not buy it? Wait before you buy. Time gives us the opportunity to really think about our goals. If we buy on impulse, we usually make poor choices. Sometimes, by waiting, the price may go down or a different or better option comes up. Look around to get the greatest deal. You might discover a different price or a better deal by doing comparison shopping. It's also possible that you'll find a less expensive, superior alternative. Seek substitutes. Is a new car really necessary? Can you afford a secondhand car? Or ride in a carpool or bus? Could you purchase from thrift stores, flea markets, or garage sales? By following a few easy steps, you can alter the way you spend and save. Modify your routines. Try reducing your weekly cigarette intake by at least two packs, for example, if you smoke.

You'll save anywhere from Rs. 10 to Rs. 15. If you commute by automobile, consider taking public transportation one or two days a week or walking there instead. You will need to raise your income if, after making every effort to reduce your expenses, you are still unable to pay for them. You could be able to work more hours or overtime with some jobs. Acquiring a part-time job over the weekends or evenings is an additional choice. Child or elder care is one example of a part-time work that can be done from home. Seek out opportunities to trade or barter with people. For instance, if you cut your friend's hair, she might watch your kids after school while you work.

If you assist your neighbor in painting their home, they might mow your grass once a week. Learning and putting new abilities to use is another strategy to boost your earnings. If you can operate a computer, for instance, you might be able to get a part-time job entering data or creating the church newsletter. You may create and sell holiday cards if you're an artist. You may make money fixing things for other people if you learn how to fix items around the house. Recall that you need to raise income and decrease expenses in order to balance your budget. Any extra cash ought to be saved.

Table 4.1 Key Abbreviations Commonly Used in Budget:

Abbreviation	Meaning	Abbreviation	Meaning
ABS	Annual Budget Statement	ADP	Annual Development Plan
AG	Accountant General	AGPR	Accountant General Pakistan Revenues
APCC	Annual Plan Coordination Committee	MTBF	Medium Term Budgetary Framework
BEs	Budget Estimates	MTDF	Medium Term Development Framework
CoA	Chart of Accounts	CBR	Central Board of Revenue
GDP	Gross Domestic Product	DDO	Drawing and Disbursing Officer
MoF	Ministry of Finance	GoP	Government of Pakistan
NIS	New Items Statement	PAC	Public Accounts Committee
PC	Planning Commission	PC-1	Planning Commission Pro-forma
P&D	Planning & Development Department	PRSP	Poverty Reduction Strategy Paper
PSDP	Public Sector Development Program	REs	Revised Estimates

4.5 Federal Budget

The federal budget's components-receipts and expenditures-are covered separately. Below is a quick summary of the Receipt component:

4.5.1 The Federal Funds Consolidation

The Federal Consolidated Fund, which is made up of both internal and external receipts, represents the entirety of government resources. Internal revenues, which come from internal resources and cover the following, comprise revenue and capital receipts:

1. Tax money that the Central Board of money (CBR) collects.
2. Tax revenue that is not collected by the CBR.
3. Non-tax income.
4. Refunds for capital.

External sources of income include grants, loans, project assistance, and the like.

4.5.2 Tax Revenue Collected by CBR

It includes the following taxes:

(a) Direct Taxes

The income and wealth of people and organizations are subject to direct taxation. These include capital gains tax, income tax, wealth tax, overseas travel tax, workers' welfare tax, and the Act of 1968's Companies Profit (Workers' Participation) payments. Every industrial company with more than 10 employees is obliged to pay 5% of their revenues to the social security administration and register with them.

(b) Indirect Taxes

These taxes, which are imposed on transactions and commodities, are as follows: sales tax, natural gas fee, petroleum development levy, federal excise, customs charges, and airport tax.

1. Customs Duties

Imported products into Pakistan are subject to customs taxes at the tariff rates specified in the First Schedule of the Customs Act 1969. Every year, the government evaluates the expected amount of revenue from this source and modifies the tax rates on all or certain imported goods into the country.

2. Sales Tax

A consumption tax known as sales tax is levied on specific services, manufacturers, importers, wholesalers, distributors, dealers, and retailers with yearly sales above Rs. 5 million.

3. Federal Excise

Federal excise duties apply to the excisable commodities mentioned in the First Schedule of the Central Excise Act 1944 that are manufactured, produced, or imported into Pakistan. Excise tax collection is one of the main sources of federal tax income. However, sales tax is progressively replacing this source of revenue and doing so in most situations. Therefore, federal excise duty only applies to nine items and one service.

ii. Tax Revenue Other Than CBR Collections

Any industrial enterprise that employs more than 10 people is required to pay 5% of their revenues toward employee profit sharing and register with the social security authorities. The residual money, after worker distribution, becomes a part of the Workers Welfare Fund and is deposited into the government account in line with the Workers Welfare Fund Ordinance 1971.

iii. Foreign Travel Tax

Foreign travel tax has been in effect since 1987 for all international plane tickets leaving from or arriving in Pakistan, including those purchased abroad. Currently, each ticket is subject to a 1,500-rupee charge. Infants, diplomats, and supernumerary crew members are not allowed.

iv. Petroleum Development Levy

Petroleum products were subject to a development surcharge under the terms of the Petroleum Products (Development Surcharge) Ordinance, 1961 and its implementing regulations. The liberalization of petroleum product pricing led to the development of the surcharge mechanism's abolition in 2001. Instead, the Finance Ordinance, of 2001 established a set petroleum fee based on the price per liter of various items that the government announces on a regular basis.

v. Surcharge on Natural Gas

The development surcharge on natural gas is established by the Natural Gas Development Surcharge Ordinance, of 1967. In compliance with this law, the federal government shall establish a price for the gas companies and decide the selling price for customers. The difference between the two prices is covered by the margin that can be used by the government as a development surcharge. Periodically, the mandatory price of the gas corporations is adjusted to reflect well-head expenditures and the cost of gas transportation and distribution. The natural gas surcharge is allocated to the provinces in accordance with their respective gas production levels.

vi. Airport Tax

Since October 1996, an airport tax has been in place. The cost of travel is determined by the class and kind of travel (domestic or international). Infants, diplomats, and supernumerary personnel are not permitted.

vii. Non -Tax Revenue

Several departments and ministries oversee the non-tax revenue of the federal government.

viii. Income from Property and Enterprise

Income from real estate and business is made up of profits, interest payments, and dividends from government assets, such as those from the Pakistan Post Office and the Pakistan Telecommunication Authority (PTA). A large amount of the account balance under this category includes interest on loans provided by the federal government to state, municipal, and federal workers as well as commercial agencies and nonfinancial enterprises.

ix. Receipts from Civil Administration and other Functions

Numerous sources, including immigration, printing, education, irrigation, courts, prisons, economic regulation, civil defense, communication and works, and forest institutions, contribute to these earnings.

x. Miscellaneous Receipts

These include economic services revenue as well as additional revenue from the explosives department, the cabinet division, the lighthouse and lightship departments, and other sources.

xi. Capital Receipts

These comprise interest income from borrowing, money received back on loans, profits from investments and advances, income from savings programs, and net receipts from activities that fall under the deposit and reserve categories. The Public Sector Development Program (PSDP) is normally funded by the net capital receipts of the federal government. Capital receipts comprise both external and internal receipts.

(a) **Internal Receipts** Internal income and receipts include, among other things, investments recovered against state-owned companies, loans and advances recovered from provinces, domestic debt receipts against government domestic bonds, and floating debt receipts against Treasury Bills, prize bonds, and other instruments.

(b) **External Receipts** primarily consist of grants under the Food Aid Convention, loans and credits from the World Food Program (WFP), friendly nations, and specialized international agencies; they also include receipts from sovereign, euro, and Sukuk bonds, project aid, and credits from bilateral and multilateral donors.

xii. Public Account

Both receipts and disbursements are included in the balances under Public Account. Receipts are deposits and reserves held in government-managed accounts as well as inflows into the government's different savings initiatives. In a similar vein, disbursements denote withdrawals from reserve accounts, deposits, and similar programs. The budget's Receivables section contains net balances.

4.6 Components of Federal Expenditure

Two categories of expenditure exist development expenditure and current (non-development) expenditure.

4.6.1 Current Expenditure

(a) This covers both current expenditures on the income account and current expenditures on the capital account.

A brief synopsis of the elements of current spending is provided below:

(b) Current expenditure on Revenue Account

1. General Public Services

This includes the expected spending plans for the legislative and executive arms of government, in addition to financial and fiscal issues, external relations, debt repayment, debt servicing, and debt, both foreign and domestic.

2. Defense Affairs and Services

Includes projected spending amounts for defense management and military defense.

3. Public Order and Safety Affairs

This includes costs for public order management, law enforcement, fire safety, prison operations, and courts.

4. Economic Affairs

Amounts are allotted for a variety of industries, including general economic, commercial, labor, agricultural, food, irrigation, forestry, and fishing; fuel and energy; mining and manufacturing; building and transportation; and communication.

5. Environmental Protection

It includes budget estimates for managing wastewater, reducing pollution, conducting environmental research and development, managing environmental protection, and more.

6. Housing and Community Amenities

It contains funding allotted to the community development, housing, and water supply departments.

7. Health

This covers funding for hospital services, public health services, medical research and development, medical appliances and equipment, and health administration.

8. Recreation, Culture, and Religion

Budget estimates for broadcasting and publishing, religious affairs, cultural services, sports, recreation, and administration of information are all included in this.

9. Education Affairs and Services

Pre-primary/primary education affairs and services, education sector reforms, tertiary education affairs and services, education services, administration, and education services not otherwise covered are all included in this.

10. Social Protection

Includes allocations for administration and others.

(c) Current Expenditure on Capital Account:

Current expenditures on the capital account include repayment of short-term foreign credits (transfers, loans, and advances) as well as other costs associated with general economic, commercial, and labor activities, such as forestry, agriculture, food, irrigation, and fisheries.

(d) Development Expenditure:
This covers both the costs associated with development in the Capital Account and the Revenue Account.

4.6.2 Development Expenditure on Revenue Account and Capital Account

The phrase "development expenditure on revenue account" refers to the amount spent on plans or initiatives for both ongoing and planned development that are financed by proceeds from routine government operations. Similarly, expenditures related to development projects financed by grants, loans, and other funding sources are referred to as development expenditures on capital account.

4.6.3 National Finance Commission (NFC) Award

Federal funds provide for the biggest annual contribution to provincial budgetary receipts. These display the share of direct and indirect taxes and charges levied and collected by the federal government on behalf of the provinces. The federal government and the provinces split the following taxes and duties:

1. Income taxes;
2. Wealth taxes;
3. Capital value taxes
4. Goods Purchase and Sales Tax
5. Cotton Export Duty
6. Customs Duty
7. Federal Excise Duty (all other taxes that the federal government may levy excluded from the Excise Duty on Gas imposed at the wellhead)
8. Any additional taxes the federal government may impose.

4.7 Self-Assessment Questions

1. Someone pays 15 pounds for a horse. He sells it for 20 pounds after a year. He buys the same horse again for 30 pounds after a year and sells it for 40 pounds. What is that person's overall earnings percentage for both deals?

2. Amir bought 160 shirts for Rs. 225 each. The cost of transportation was Rs. 1,400. He had to pay Rs. 1.75 per shirt for an octroi and Rs. 320 for the work. How much should he charge for one shirt if he wants to make 20%?

3. A man sold two homes for a total of Rs. 7.8 lakhs. He made 5% on one and lost 5% on the other. How much of an impact does the sale have on the whole?

4. A store owner paid Rs. 500 for each of the 150 pen drives he got. It cost him 500 rupees to get there and pack. If a pen drive is priced at Rs. 520 and a shopkeeper gives a 5% discount on that price, how much profit does the shopkeeper make?

5. A 25% profit is made on an item sold. The profit percentage drops by 15% if the article's C. P. and S. P. are raised by Rs. 60 and Rs. 30, respectively. Determine the article's cost.

6. In this case, the person who sold the pen for Rs. 96 made the same amount of money as the watch's cost price. If he sells it for twice as much as the percentage profit he made on it before, what will the new price be?

7. If the cost price of 12 pens is the same as the price of 8 pens, find the percentage gain.

8. Three cricket balls cost the same as two pairs of leg pads. Three pairs of leg pads cost the same as two pairs of socks. Three pairs of gloves cost the same as two cricket bats. How much is a cricket ball if a bat costs Rs 54?

9. If the sell price is 25% more than the cost price. Then how much of a profit is there?

10. Asif got 30% off the item's listed price and sold it for Rs. 8750, making a 25% return on the price he paid for it. Find the price that's written on the box.

11. A water cooler is listed at Rs. 4650. The store owner gives an 18% discount on it during the off-season. Find out how much it's worth.

12. A shop owner cut the price of a sweater from Rs. 960 to Rs. 816 during the winter. Find the savings rate he offered.

13. Figure out how much of a discount is being given on a shirt that costs Rs. 546 after a Rs. 104 discount is taken off of its marked price.

14. An 8% discount is put on a toy, and it is then sold for Rs. 216.20. Look at the toy's price tag.

15. A tea set was bought for Rs. 528, which is 12% less than what it was originally priced at. Find the tea set's listed price.

16. A store owner puts a price on his goods that is 35% higher than what it costs to make them, but an extra 20% is taken off of that price. Find his percent gain or loss.

17. Mr. Mujahid possesses 15 tola gold and 10 tola silver. Calculate Zakat, use current market values of gold and silver per tola in rupees.

18. Mr. Irfan owned a factory worth Rs. 3 crores. He gained a profit of Rs. 3456780 last years and possesses 4 tola gold and 9 tola silver. Calculate Zakat, and use current market values of gold and silver per tola in rupees.

19. For two years and six months, how much simple interest can someone get on Rs. 8,200 if the interest rate is 17.5% per year?

20. How long will it take for Rs. 4,000 lent at 3% per year on simple interest to make the same amount of interest as Rs. 5,000 lent at 4% per year on simple interest in 5 years?

21. Mr. Qasim put Rs. 13,900 into two different plans, A and B, which paid 14% p.a. and 11% p.a. in simple interest, respectively. If the simple interest gained over two years adds up to Rs. 3,508, how much was put into Scheme B?

22. At a simple interest rate of 4.5% per year, how long will it take for Rs. 450 to earn Rs. 81 in interest?

23. A bank gives 5% interest that is calculated every six months. A customer puts down Rs. 1600 on January 1 and July 1 of each year. Figure out how much he would have gained in interest by the end of the year.

24. At 7% per year, the interest on Rs. 30,000 is Rs. 4347. Figure out the time (in years).

25. How much interest will be added to Rs. 25,000 every year for three years if the rate is 12% per year?

26. Mr. Afzal put Rs. 8,000 into a fixed savings plan that would earn 5% p.a. interest over two years. In the end, how much money will Albert get from the set deposit?

Unit–5

MATHEMATICAL EXPRESSIONS

Written by: Dr. Muhammad Nazam

Reviewed by: Dr. Muhammad Asad Zaigham

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5.1 Introduction

We will study mathematical expressions in this unit, including how to equate two expressions in a single variable and apply this knowledge to address real-world issues. The Golden Ratio and its applications in social interactions will be covered. We will study statement comparisons and how to apply them to solve social and economic issues. We'll study the idea of number patterns and how they're used.

5.2 Objectives

After the successful learning of this unit, students will be able to;

- Understand and solve linear and quadratic equations
- Understand mathematical modeling
- Understand and solve inequalities
- Understand golden ratio
- Understand number patterns

5.3 Mathematical Expression

In algebra, we come across four expressions algebraic expression, trigonometric expression, logarithmic expression, and exponential expression. If an expression is composed of two or more different mathematical expressions, then it is known as a transcendental expression. An equation is formed if we connect two mathematical expressions with the equality sign "=". The expression involving one or more variables having a maximum exponent of 1 is known as linear expression. The equation of two linear expressions is known as a linear equation. The general form of a linear equation is $ax + by + c = 0$; $a, b,$ and c are known real numbers. The equation in which the maximum exponent is two is known as a quadratic equation. The general form of a quadratic equation in variable x is $ax^2 + bx + c = 0$; a, b, c are known real numbers. The general form of a cubic equation in variable x is $ax^3 + bx^2 + cx + d = 0$; a, b, c, d are known real numbers. The polynomial is an expression in which exponents of variables are non-negative integers.

5.3.1 Linear Equations in One Variable and Word Problems

Numerous issues can be expressed as equations and include relationships between known and unknown numbers. We refer to these problems as word problems since the equations are typically expressed in words. We will address a few real-world issues using equations in one variable.

Steps involved in solving a linear word problem:

- Examine the situation carefully, noting what is supplied and what is required. Make use of the variables x, y, and so on to indicate the unknown. Convert the issue into mathematical terminology or mathematical assertions.
- Using the conditions listed in the problems, create a linear equation in a single variable.
- Find the unknown by solving the equation.
- Confirm that the solution meets the requirements of the problem.

Example: The two numbers add up to 25. There is a nine-figure difference between the two. Locate the numerical values.

Solution: Let the number be x. Then the other number $= x + 9$. The sum of two numbers = 25 According to question, $x + x + 9 = 25 \Rightarrow 2x + 9 = 25 \Rightarrow 2x = 25 - 9 \Rightarrow 2x = 16 \Rightarrow 2x/2 = 16/2$ (divide by 2 on both the sides) $\Rightarrow x = 8$ and other number is $x + 9 = 8 + 9 = 17$.

The required two numbers are 8 and 17.

Example: There is a 48-bit discrepancy between the two numbers. The two numbers have a 7:3 ratio. Which two numbers are they?

Solution: Let the common ratio be x. Their difference = 48. According to the question,

$$7x - 3x = 48 \Rightarrow 4x = 48 \Rightarrow x = \frac{48}{4} \Rightarrow x = 12.$$

First number $= 7x = 7 \times 12 = 84.$

Second number

$$= 3x = 3 \times 12 = 36.$$

Example: A rectangular field has a surface size of 300 square meters. Its circumference is seventy meters. Determine this rectangle's width and length.

Solution: Let L is the length and W is the width. By given conditions

Area=L × W = 300. -----(1)
Perimeter $= 2L + 2W = 70$, this implies that $L = 35 - W$. Putting this value of L in (1), we have $(35 - W) \times W = 300 \Rightarrow 35W - W^2 = 300$ *or* $W^2 - 35W + 300 = 0$. Now, $W^2 - 35W + 300 = W^2 - 15W - 20W + 300 = W(W - 15) - 20(W - 15) = 0$. This implies that $(W - 15) = 0$ or $(W - 20) = 0$. Thus, $W = 15$ or $W = 20$ and hence, $L = 35 - W = 35 - 15 = 20$ or $L = 35 - W = 35 - 20 = 15$.

Example: Four shirts, four sets of pants, and two hats cost $560 in a store. Nine shirts, nine pairs of pants, and six hats total $1,290 in cost. How much does one shirt, one pair of pants, and one that cost all together?

Solution: Let y be the cost of a single pair of pants, z be the cost of a single hat, and x be the cost of a single shirt. By stated conditions, we have

$$4x + 4y + 2z = 560. ----- (1)$$
$$9x + 9y + 6z = 1{,}290. \quad (2)$$

By (2) ÷3, we have,

$$3x + 3y + 2z = 430. ---- (3)$$

By $(1) - (3)$, we have

$$x + y = \$130.$$

By (3), we have $3(x + y) + 2z = 390 + 2z = 430$. This implies $z = \$\,20$.

Example: There are four kids with tiny toys. The number of toys belonging to the first child is one-tenth that of the second, the third child has one more toy than the first, and the fourth child has twice as many as the third. What is the quantity of toys?

Solution: Let the total No. of toys be x, so that

No. of toys for first child = $x/10$,

No. of toys for second child = $x/10 + 12$,

No. of toys for the third child = $x/10 + 1$ and

No. of toys for the fourth child = $2(x/10 + 1)$.

By the stated condition, we have

$$\frac{x}{10} + \frac{x}{10} + 12 + \frac{x}{10} + 1 + 2\left(\frac{x}{10} + 1\right) = x.$$

Solving the above equation, we have $x = 30$ toys.

Example: On an exam, a class average is 70. Students who earned less than 60 on average had a score of 50. 75 is the average score for students who received 60 or higher. How many pupils in this class received a score lower than 60 if there were 20 in total?

Solution: Assign x to all the pupils who received a score of less than 60 and y to all those who received a score of 60 or higher. Let A stand for grades less than 60 and B for grades sixty or higher. As per the specified terms, we have

$$\frac{\sum A + \sum B}{20} = 70. - - - - - (1)$$

$$\frac{\sum A}{n} = 50. - - - - - - - \quad (2)$$

$$\frac{\sum B}{m} = 75. - - - - - - - - \ (3)$$

By (1), (2) and (3), we have

$$50n + 75m = 1400 - - - - - - (4)$$

$$n + m = 20. - - - - - - - - - - - (5)$$

Solving (4) and (5) simultaneously, we have $n = 4$ and $m = 16$.

Example: A boat needs three hours to go from point A to point B down a river, and five hours to go up the river from point B to point A. In calm water, how long would it take the identical boat to get from point A to point B?

Solution: Let d be the distance between A and B, x be the rate of the water current, and S be the boat's speed in calm water. Under the given circumstances, we have,

$d = 3(S + x)$.
$d = 5(S - x)$.

This implies that

$$3(S + x) = 5(S - x) - - - -(1).$$

Solving (1) for x, we have $x = S / 4$.
Substituting the value of x in $d = 3(S + x)$, we have

time$= d / S = 3.75$ hours = 3 hours and 45 minutes.

Example: In eight hours, an airplane travels from point A to point B against the wind. In seven hours, the same aircraft makes its way back from B to A in the direction of the wind. Determine the aircraft's speed in relation to the wind speed while the air is still.

Solution: Let S represent the airplane's speed in still air, x represents the wind's speed, and d represent the separation between A and B. Given the circumstances, we have

$$d = 8(S - x).$$
$$d = 7(S + x).$$

Thus,

$$8(S - x) = 7(S + x).$$

The required ratio is $= S/r = 15$.

5.4 Golden Ratio

In mathematics, two quantities x and y (; $x > y$) are in the golden ratio **(**denoted by φ) if the ratio between them and the greater of the two values is the same. Algebraically, we say that x and y (taking $x > y$) form a golden ratio, if,

$$\frac{x+y}{x} = \frac{x}{y}. - - - - -(1)$$

By (1), we have

$$1 + \frac{y}{x} = \frac{x}{y}, \text{this implies that } 1 + \frac{1}{\varphi} = \varphi.$$

On further simplifications, we have $\varphi^2 - \varphi - 1 = 0$. By quadratic formula, we obtain:

$$\varphi = \frac{1+\sqrt{5}}{2} = 1.618033988749 \ldots.$$

This shows that the Golden Ratio is a fixed irrational number. It is the ratio of the diagonal of a regular pentagon to its side. It is sometimes referred to as the divine proportion, the golden number, or the golden ratio. It has a close relationship to the Fibonacci sequence, which is a set of integers where each one is added to the previous one. The ratio between each Fibonacci number and the preceding number steadily approaches 1.618. The Fibonacci numbers are 0,1,1,2,3,5,8,13,21, and so forth.

Example: If the Golden Ratio is 1.618 and you have a piece of cardboard that is $\frac{7}{8}$ ft wide. How long should you cut, so, it follows the Golden Ratio?

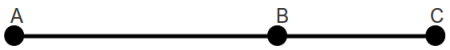
Solution. Let the length of the cardboard be x, by given condition, we have,

$$1.618 = \frac{x}{0.875} \Rightarrow x = 1.416.$$

Problem 1 (Finding the Golden Ratio)

Assume the line segment below satisfies the following:

$$\frac{AB}{BC} = \frac{AC}{AB}$$



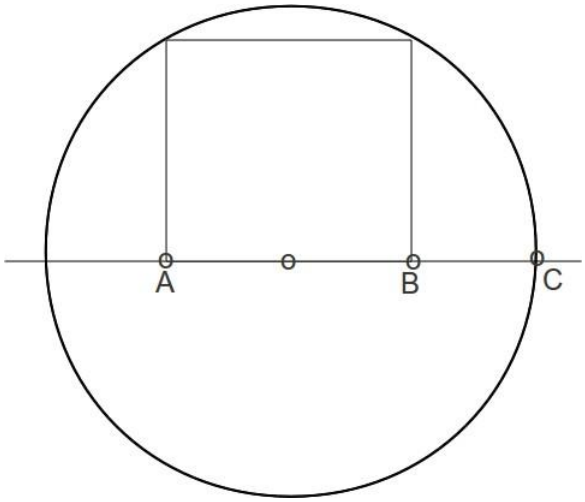
(a) What proportion is this? (Hint: Solve for x by setting AC = 1, AB = x, and BC =? How come we can set AC to 1?). Is it possible to calculate the ratio without presuming that AC = 1?

(b) In section (a), what quadratic equation did you solve? Is there another comparable quadratic equation whose reciprocal of x is its solution?

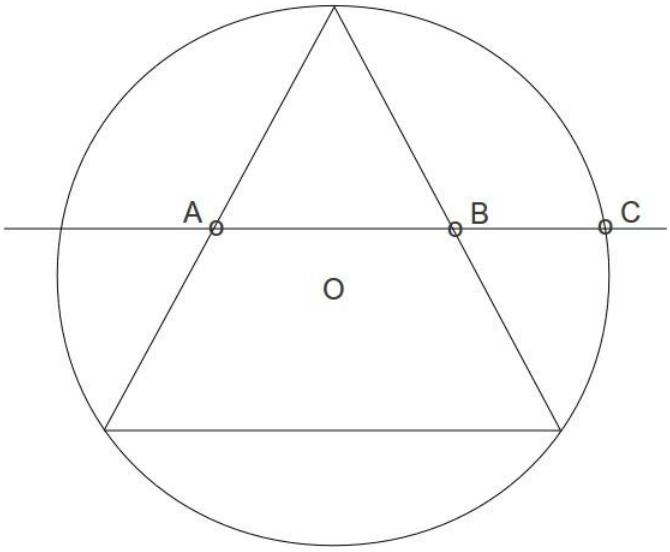
Problem 2 (Golden Ratio in Geometry)

There are some basic geometric constructions that use the Golden Ratio.

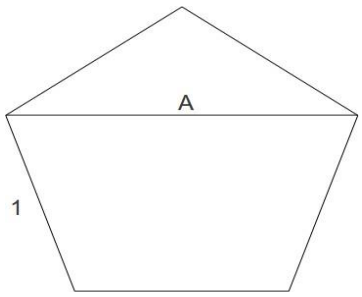
(a) A semicircle contains an inscribed square. The ratio $\frac{AB}{BC}$ is the golden ratio. Prove it.



(b) An equilateral triangle is carved in a circle. We draw a line through the middle of two of the triangle's sides. The golden ratio is AB divided by BC. Show it.



(c) A regular pentagon has five equal-length sides and interior angles of 108^°. Line segment A satisfies the golden ratio. Display it.



Problem 3 (Rabbits that Never Die)

Once a month, a pair of adult rabbits gives birth to a pair of baby bunnies. It takes one month for every pair of newborn bunnies to mature into adults, and then every month after that, they give birth to another pair of babies. After a certain number of months, count the number of pairings consisting of adult and baby rabbits. Rabbits never die.

Problem 4 (Golden Ratio and Fibonacci Sequence)

Let $a_0 = 0$ and $a_1 = 1$. The general term of the Fibonacci sequence is given by:

$$a_{n+2} = a_{n+1} + a_n.$$

The first few numbers of the sequence are:

$$0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, \ldots$$

(i) Recall that the golden ratio φ is a solution to $\varphi^2 - \varphi - 1 = 0$.

Develop a relation between φ^n and φ where *n* is a positive integer. (Hint: So, we know $\varphi^2 = \varphi + 1 \Rightarrow \varphi^3 = \varphi^2 + \varphi \Rightarrow \varphi^3 = 2\varphi + 1$. Now generalize to φ^n. Your answer will involve the Fibonacci sequence).

(ii) Denote $\frac{1}{\varphi}$ by ρ. Recall that $\rho = \frac{-1+\sqrt{5}}{2}$ and it satisfies the following equation:

$$(-\rho)^2 = (-\rho) + 1.$$

Is it clear that $(-\rho)$ satisfies the same relation as φ in part (a)?

(iii) Use parts (a) and (b) to simplify the expression $\varphi^n - (-\rho)^n$. You should get an expression involving a_n. Solve for a_n. Find $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \varphi$.

(iv) Using the expression a_n obtained in (iii), show that,

5.4.1 Comparison of Statements and Their Use in Social and Economic Problems

A comparison statement is just a statement that compares two values or quantities. For example,

(i) Mary and Sally are of the same height, or

(ii) if x apples are added to 3 apples, the total number of apples will not exceed 10.

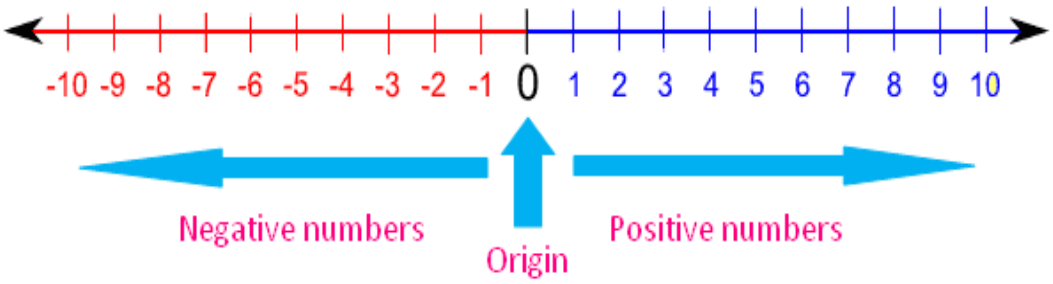
In mathematics, comparison operators, variables, and integers are used while writing a comparison statement. Comparing two quantities so that one multiplied by a given number yields the other is known as a multiplicative comparison. Sam, for instance, has two times as many balloons as Sid. Word issues that need multiplicative comparison can be resolved by formulating a multiplication equation. Phrases that are typically found in multiplicative comparison problems:

- Five Times greater than (Jake has five times more stamps than Ben).
- Times as much-my sister makes three times my salary.
- Three Times longer (three times more than a foot is a yard).
- Exponent of height (the building is four times the tree's height).

The symbols used for comparing numbers in the statements are, " $>$ " which means greater than, " $<$ " which means less than, and " $=$ ", which means equal to.

Comparing the numbers on a number line is very easy. We just need to know the facts given below:

- The number on the right is greater for any two numbers marked on the number line, and the number on the left is smaller for any two numbers marked on the number line.



Note that by comparing the two statements, we form an inequality. In the following, we learn about two fundamental inequalities and their solutions.

Polynomial Inequality: Any inequality that can take one of the following forms is a polynomial inequality:

$$f(x) < 0, f(x) > 0, f(x) \leq 0, \text{or } f(x) \geq 0,$$

in which the function f is a polynomial. For instance, a quadratic inequality or polynomial inequality of degree two is the condition ax^2+ bx + c>0. A method for comparing two mathematical assertions is polynomial inequality.

Solving Polynomial Inequality:
Finding the solution set for polynomial inequalities with the form $f(x) < 0$ or $f(x) > 0$ often involves locating the x-intercepts of the polynomial function, f. The boundary points that split the real number line into intervals are the x-intercepts of f. The graph of f is either below the x-axis $[f(x) < 0]$ or above it $[f(x) > 0]$ on each interval. Because of this, x-intercepts are essential to the solution of polynomial inequalities. One can determine the x-intercepts by resolving the equation $f(x) = 0$. An interval is the only solution to polynomial inequality.

How Polynomial Inequalities Are Solved:

1. Write the inequality in terms of polynomial functions, f, such that $f(x) > 0$ or $f(x) < 0$.

2. Find the solution to $f(x) = 0$. The boundary points are the actual solutions.

3. To create intervals on a number line, find these boundary points on the line.

4. Within each interval, pick one sample number, referred to as a test value, and assess f at that value.
   (i) For all values, x, in the interval, $f(x) > 0$ if the value of f is positive.
   (ii) For all values, x, in the interval, $f(x) < 0$ if the value of f is negative.

5. Determine which interval, if any, fulfills the given inequality and record the set of solutions. If > is substituted with ≥ or < is substituted with ≤, then this process is legitimate. On the other hand, add the boundary points [the solutions of $f(x) = 0$] in the solution set if the inequality contains ≤ or ≥.

Example: Compute the solution set and plot it on a real number line: $2x^2 + x > 15$.

Solution:

Step-1: (Write the inequality in the following format: $f(x) < 0$ or $f(x) > 0$).
The inequality is first rewritten using 0 on the right side.

$$2x^2 + x > 15;$$
$$2x^2 + x - 15 > 15 - 15;$$
$$2x^2 + x - 15 > 0.$$

The inequity we are trying to fix is the same as this one. The format is $f(x) > 0$, with $f(x) = 2x^2 + x - 15$.

Step-2: (Solve the equation $f(x) = 0$).

We find the *x*-intercepts of $f(x) = 2x^2 + x - 15$ by solving the equation $2x^2 + x - 15 = 0$.

$$2x^2 + x - 15 = 0;$$
$$(2x - 5)(x + 3) = 0;$$
$$2x - 5 = 0 \text{ or } x + 3 = 0;$$

$x = 52$ or $x = -3$.
f has two x-intercepts: -3 and 52. These x-intercepts will serve as the boundary points on a number line that we use.

Step-3: Draw a number line, mark the border points, then divide the line into intervals.

The number line is divided into three intervals by the boundary points:
$(-\infty, -3), \left(-3, \frac{5}{2}\right), \left(\frac{5}{2}, \infty\right)$

Step-4: (Within each interval, select one test value, and calculate f at that value). **Interval**	**Test Value**	**Substitute into** $f(x) = 2x^2 + x - 15.$	**Conclusion**
$(-\infty, -3)$	-4	13	$f(x) > 0, \forall\, x \in (-\infty, -3)$
$(-3, 5/2)$	0	-15	$f(x) < 0, \forall\, x \in (-3,5/2)$
$(5/2, \infty)$	3	6	$f(x) > 0, \forall\, x \in (5/2, \infty)$

Step-5: Select the interval or intervals that meet the given inequality and record the set of solutions.

We are interested in solving $f(x) > 0$, where $f(x) = 2x^2 + x - 15$. Based on our work in step 4, we see that:

$$f(x) > 0, \forall\, x \in (-\infty, -3) \text{ and } \left(\frac{5}{2}, \infty\right).$$

Consequently, the given inequality's solution set, $2x^2 + x - 15 > 0$, is:

$$\left\{x \mid x \in (-\infty, -3) \cup \left(\frac{5}{2}, \infty\right)\right\}.$$

The following is the graph of the solution set on a number line:

Example: With an initial velocity of 96 feet per second, a ball is hurled vertically upward from the top of the Leaning Tower of Pisa, which is 190 feet high. How long will it take for the ball to go higher than the tower?

Solution: Let v_0 represents initial velocity and $s_0$represents initial position, then the position function for a free-falling object is given by

$$s(t) = -16t^2 + v_0 t + s_0. - - - - - - - (1)$$

We are given that $v_0 = 96$ and $s_0 = 190$, so that (1) implies that:

$$s(t) = -16t^2 + 96t + 190. - - - - - - - (2)$$

By condition, we need to find those values of t, which satisfy the following inequality:

$$-16t^2 + 96t + 190 > 190.$$

Proceeding as in Example 1, we have the following solution set that is the time range in which ball's height exceed that of the tower:

$$\{t \mid t \in (0,6)\}.$$

Rational Inequality: A rational inequality is any inequality that can be put into one of the forms:

$$f(x) < 0, f(x) > 0, f(x) \leq 0, \text{or } f(x) \geq 0,$$

where f is a rational function. For example, the inequality $\frac{3x^2}{x+1} > 0$ is a rational inequality.

Instructions for the solution of rational inequality:
To begin solving a rational inequality, move all of the terms to one side and get zero on the other. Next, represent the rational function as a single quotient on the nonzero side. Setting the rational function f's numerator and denominator to zero is the second step. The real number line is divided into intervals by the boundary points formed by the solutions to these equations. The process is now the same as the one we applied to solve polynomial inequality.

Example: Solve and graph the solution set of the following rational inequality:

$$\frac{x+1}{x+3} \geq 2.$$

Solution:
We have

$$\frac{x+1}{x+3} \geq 2 \Rightarrow \frac{x+1}{x+3} - 2 \geq 0 \Rightarrow \frac{-x-5}{x+3} \geq 0.$$

It is in the form $f(x) \geq 0;\ f(x) = \frac{-x-5}{x+3}$.

Zero should be set for both the numerator and the denominator of f. The boundary points: $x = -3,\ \ x = -5$ represent the actual solutions. The remaining part of the solution is like those polynomial inequalities.

5.5 Number Patterns and their Applications

Numerous spirals and other complex mathematical patterns can be found in all of nature's creations. For instance, spirals are formed when the individual florets in a sunflower's head are arranged. There are 21 spirals in a clockwise direction and 34 spirals in a counterclockwise way in some species. According to the type of sunflower, the exact numbers can be 21 and 34, 34 and 55, 55 and 89, or even 89 and 144. This finding is made even more intriguing when we take into account a pattern or series of numbers that the thirteenth-century Italian mathematician Leonardo of Pisa, popularly known as Fibonacci, studied.

The Fibonacci sequence of numbers starts as follows and is unlimited in length: 3, 5, 8, 13, 21, 34, 55, 89, 144, 233... The initial pair of terms are 1. Any additional terms are the total of the two terms that came before them. For instance, the first and second terms add up to the third term, 2, which is 1 + 1 =2. The second and third terms add up to the fourth term, which is 3, as follows: 1 + 2 = 3. Were you aware that there are two Fibonacci numbers, 21 and 34, representing the number of spirals in a daisy or a sunflower? Fibonacci numbers also apply to the number of

spirals in a pineapple (8 and 13) and a pinecone (8 and 13). The terms of the Fibonacci sequence:

$$1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, \ldots$$

are the range values for a function f:$\mathbb{N} \to \mathbb{R}$. Thus, $f(1) = 1, f(2) = 1, f(3) = 2, f(4) = 3, f(5) = 5, f(6) = 8, f(7) = 13$, and so on. Instead of using the standard function notation, function values in a sequence are represented by the letter a with a subscript. The subscripts denote a term's placement and comprise the sequence's domain. The first term of the sequence is represented by $a_1 (a_1 = \text{f}(1))$, the second term by a_2 $(a_2 = \text{f}(2))$, the third term by a_3, and so on. The nth term, or general term, of a sequence, is represented by the notation a_n. $\{a_n\}$ is a representation of the entire sequence. The Fibonacci sequence's nth term is provided by:

$$a_n = a_{n-1} + a_{n-2,} \text{ for all } n > 1.$$

Because it appears regularly in nature and has applications in a variety of fields, such as mathematics, science, art, and finance, the Fibonacci Sequence is important. Examples of sequences found in nature include the way leaves are arranged on stems, the way trees branch, and the spiral patterns found in shells and galaxies. It is also used to characterize patterns of population increase and stock market trends.

Example: If φ is the golden ratio, and a_{15} is 610, what is a_{16} in the Fibonacci sequence?

Solution:
We know that for a Fibonacci sequence, $a_n \cong a_{n-1} \times \varphi$. Thus, if a_{15} is 610, then,

$$a_{16} = 610 \times 1.618 = 986.98 \cong 987.$$

Activity 1: There are four more significant number patterns, given in the following:

(i) The Lucas Numbers.

(ii) Yellowstone Permutation Integer Sequence.

(iii) Padovan Sequence.

(iv) Perrin Sequence.

Write down their details and general term formulas.

Activity 2: Define the functions $f: \mathbb{N} \to \mathbb{R}$ and $g: [0, \infty) \to \mathbb{R}$ by $a_n = f(n) = \frac{1}{n}$ for all $n \in \mathbb{N}$, and $b_n = g(m) = \frac{1}{m}$ for all $m \in [0, \infty)$. Compare the graphs of the sequence $\{a_n\}$ and the function g.

Example: Define the functions $f, g: \mathbb{N} \to \mathbb{R}$ by

$$a_n = f(n) = 3n + 4 \quad \text{for all } n \in \mathbb{N};$$

$$b_n = g(n) = \frac{(-1)^n}{3^n - 1} \quad \text{for all } n \in \mathbb{N}.$$

Write down the first ten terms of the number patterns denoted by $\{a_n\}, \{b_n\}$.

Solution: We put $n = 1,2,3,4,5,6,7,8,9,10, \ldots$, in the general terms (nth terms) of sequences $\{a_n\}, \{b_n\}$ defined above. We see that $a_{1=}f(1) = 3(1) + 4 = 7$, and so on, similarly, $b_{1=}g(1) = \frac{(-1)^1}{3^1-1} = \frac{-1}{2}$, and so on.
Thus,

$$\{a_n\} = \{7,10,13,16,19,22,26,29,32,35, \ldots\},$$

$$\{b_n\} = \left\{\frac{-1}{2}, \frac{1}{8}, \frac{-1}{26}, \frac{1}{80}, \frac{-1}{242}, \frac{1}{728}, \frac{-1}{2186}, \frac{1}{6560}, \ldots\right\}.$$

There are two more very important number patterns known as
(i) Arithmetic number patterns or sequences
(ii) Geometric number patterns or sequences

Arithmetic sequence: Arithmetic sequence is a number pattern with the same common difference between every two consecutive terms. For instance, the common difference in the arithmetic sequence 3, 9, 15, 21, and 27 is 6. An arithmetic progression is another name for an arithmetic sequence.

Geometric sequence: Geometric sequence is a number pattern with the same common ratio between every two consecutive terms. For instance, the common ratio in the geometric sequence 3, 9, 27, 81, … is 3. Geometric progression is another name for a geometric sequence.

Let d represents the common difference and $\{a_n\}$ represents the arithmetic sequence, then the general term formula for arithmetic sequence is given by: $a_n = a_1 + (n-1)d$. Suppose that r is the common ratio of the geometric sequence $\{b_n\}$, then the general term formula is given by: $b_n = b_1 r^{n-1}$ for all positive integers n. The arithmetic number pattern does not converge whereas the geometric number pattern converges if $-1 < r < 1$.

Activity 3: Determine whether each sequence is arithmetic, geometric or neither. If it is arithmetic, determine the constant difference. If it is geometric determine the constant ratio.

1) $\{18,22,26,30,34, \ldots\}$ 2) $\{9,19,199,1999, \ldots\}$

3) $\{8,12,18,27, \ldots\}$ 4) $\{15,7,-1,-9,-17, \ldots\}$

5) $\{12,23,34,45,56,\ldots\}$ 6) $\{100,-50,25,-12.5,\ldots\}$

7) $\{-8,12,32,52,\ldots\}$ 8) $\{1,4,9,16,25,\ldots\}$

9) $\{11,101,1001,10001,\ldots\}$ 10) $\{12,15,18,21,24,\ldots\}$

11) $\{80,20,5,1.25,\ldots\}$ 12) $\{5,15,45,135,405,\ldots\}$

13) $\{1,3,6,10,15,\ldots\}$ 14) $\{2,4,6,8,10,\ldots\}$

15)$\{-1,-2,-4,-8,-16,\ldots\}$ 16) $\{1,1,2,3,5,8,13,21,\ldots\}$.

Example: Find the nth term of the sequence $\{24,19,14,9,4,-1,-6,\ldots\}$.

Solution: We note that the given sequence is arithmetic having a common difference (-5), so, general term $= 24 - 5(n-1) = -5n + 29$.

Example: Find the nth term of the sequence $\{2,6,18,54,162,\ldots\}$.

Solution: We note that the given sequence is geometric having a common ratio (3), so, general term $= 2 \times 3^{n-1}$.

5.6 Self-Assessment Questions

Solve and graph the solution set of the following inequalities on a real number line.

1. $4x^2 \leq 1 - 2x$.
2. $9x^2 + 3x - 2 \geq 0$.
3. $3x^2 + 16x < -5$.
4. $x^2 \leq 4x - 2$.
5. $\frac{x+1}{x+3} < 2$.
6. $\frac{3x+5}{6-2x} \geq 0$.
7. $\frac{x+4}{2x-1} \leq 3$.

Find the general term formula for the following number patterns.

8. 2,5,10,17,26,…
9. 0,2,5,9,14,20,…
10. 8,12,17,23,30,…
11. 1,5,23,119,719,…

Find the first five terms of the number patterns given by.

12. $a_1 = 11 \; and \; d = 7.$

13. $a_1 = 18 \; and \; d = 9.$

14. $a_1 = -7 \; and \; d = 4.$

15. $a_1 = -8 \; and \; d = 5.$

16. $a_1 = 14 \; and \; d = -9.$

17. $a_1 = -3 \; and \; d = -3.$

In the following exercises, determine if each sequence is arithmetic, and if so, indicate the common difference.

18. 4,12,20,28,36,44, ...

19. $-7, -2,3,8,13,18, ...$

20. $-15, -16, 3, 12, 21, 30, ...$

21. $11, 5, -1, -7, -13, -19, ...$

22. $8, 5, 2, -1, -4, -7, ...$

In the following, the general term of the sequence is given.

23. Is the sequence geometric or arithmetic?

24. State the first five terms.

25. Find the value a_{18}.

26. $a_n = 6.\, 2^{n-1}.$

27. $a_n = 52 - 13n.$

28. $a_n = 5n.$

29. $a_n = 5\left(\frac{2}{3}\right)^n.$

30. $a_n = 2 + \frac{n}{4}.$

31. $a_n = 3(-3)^{n-1}.$

32. $a_1 = 2.5, r = 4.$

33. m$a_1 = \frac{1}{8}, r = \frac{1}{2}.$

34. Ava's mother is 3 times as old as Ava. Her grandmother is twice as old as Ava's mother. The sum of their three ages is 120. How old are Ava, her mother, and her grandmother?

35. The sum of the three numbers is 98. The ratio of the first to the second is $2/3$, and the ratio of the second to the third is $5/8$. Find the second number.

36. A salesman sold twice as many pears in the afternoon than in the morning. If he sold 360 kilograms of pears that day, how many kilograms did he sell in the morning and how many in the afternoon?

37. Mary, Peter, and Lucy were picking chestnuts. Mary picked twice as many chestnuts as Peter. Lucy picked 2 kg more than Peter. Together the three of them picked 26 kg of chestnuts. How many kilograms did each of them pick?

38. The sum of the two numbers is 27 and their product is 50. Find the numbers.

39. If the area of a rectangle is 84 sq units and the longer side is 5 units more than the shorter side. Find the length of both sides.

40. From A to B, Aryan travels 1 km. He travels back at a pace of three kilometers per hour faster than he did in the beginning. The travel takes 0.5 hours in total. Determine the pace for the initial and subsequent portions of the trip.

41. If the cost of bananas is increased by Re. 1 per dozen, one can get 2 dozen less for Rs. 840. Find the original cost of one dozen bananas.

42. A train travels 360 km at a uniform speed. If the speed had been 5 km/h more, it would have taken 1 hour less for the same journey. Find the speed of the train.

43. The speed of a boat in still water is 15 km/hr. It goes 30 km upstream and returns downstream to the original point in 4 hours 30 minutes. Find the speed of the stream.

Unit–6

IMPORTANCE OF GEOMETRY IN ARCHITECTURE & LANDSCAPE

Written by: Dr. Muhammad Nazam

Reviewed by: Dr. Muhammad Asad Zaigham

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6.1 Introduction

We will study geometrical things in this unit through the lens of architecture and landscape, with a focus on the social and economic challenges surrounding geometrical objects. In both Islamic and non-Islamic settings, geometry is a crucial component that is shared. There were other Greek men who were interested in mathematics besides Pythagoras. Later, a man by the name of Euclid expanded on the discoveries made by the ancient Pythag, essentially creating what is taught in geometry classes today.

6.2 Objectives

After the successful learning of this unit, students will be able to:

- Recognize the geometrical shapes that are being used in Architecture and landscapes
- Understand the points, line, ray, line segment, plane, space, and angle
- Calculate the area, perimeter, and volume of different geometrical shapes
- Understand the use of turn, flip, and slide in Architecture
- Understand the use of notions of symmetry and congruence in geometrical shapes

6.3 Introduction to Geometrical Objects Through Architecture and Landscape

Architecture and landscape design often incorporate various geometrical objects to create visually appealing and structurally sound spaces. These objects, derived from fundamental geometric shapes, contribute to the overall aesthetic appeal and functionality of the built environment. Let's explore some common geometrical objects found in architecture and landscape design:

6.3.1 Cubes

Cubes are solid three-dimensional objects with six square faces. In architecture, cubes are often used to create modern, minimalist structures. For example, some contemporary homes feature cubic forms that emphasize simplicity and clean lines.

6.3.2 Spheres
Spheres are perfectly round, three-dimensional objects. They can be seen in architectural features such as domes or rotundas. Spherical elements add a sense of grandeur and elegance to buildings, creating a visually striking focal point.

6.3.3 Cylinders
Cylinders are geometric objects with two parallel circular bases connected by a curved surface. They are commonly found in architecture in the form of columns or pillars. Cylindrical structures provide strength and support while adding a sense of verticality and symmetry.

6.3.4 Pyramids
Pyramids are polyhedrons with a polygonal base and triangular faces that converge at a single point called the apex. Architectural structures inspired by pyramids, such as the iconic pyramids of Giza, showcase their association with strength, stability, and cultural significance.

6.3.5 Rectangles and Squares
Rectangles and squares are fundamental shapes frequently used in architecture and landscape design. They form the basis for floor plans, building facades, and outdoor spaces. Their right angles and symmetrical proportions create a sense of balance and order.

6.3.6 Curves
Curves, although not strictly geometric objects, play a vital role in architecture and landscape design. They can be found in arches, bridges, or winding pathways. Curved elements introduce a sense of fluidity, movement, and organic aesthetics to the built environment.

6.3.7 Triangles
Triangles are three-sided polygons. They are often employed in architectural structures, such as gables or roof trusses, to provide stability and distribute weight efficiently. Triangles also convey a sense of dynamism and visual interest.

6.3.8 Polygons
Polygons are two-dimensional shapes with multiple sides. In architecture, polygons can be seen in the design of windows, skylights, and facades. Irregular polygons, such as hexagons or octagons, create unique patterns and add complexity to architectural compositions.

By incorporating these geometrical objects into architectural and landscape designs, designers and architects not only create visually captivating spaces but also ensure structural integrity and functional efficiency. These objects contribute to the overall composition, aesthetic appeal, and user experience, transforming the built environment into an engaging and harmonious setting.

6.4 Basic Notions in Geometry

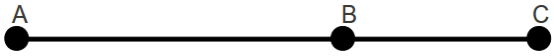
We must first comprehend the composition of geometry in order to appreciate its significance in landscape design and architecture. In mathematics or in real life, geometry is the study of various forms, figures, and sizes. Geometry teaches us about figures' similarities, transformations, and various angles. The fundamental concepts of geometry primarily rely on points, lines, angles, and planes. These fundamental ideas in geometry serve as the foundation for all geometric shapes. The following ideas are crucial to the study of geometry:

Point, the location of any object on the earth is known as a point.

Line is a one-dimensional shape that stretches infinitely in both directions and has length but no width.



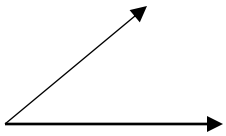
Line segment is a segment of a line with well-defined ends. Line segments AB and BC are shown in the accompanying figure.



Ray is a segment of a line that extends infinitely in one direction and has a single endpoint or starting point.



An angle is a shape created by two rays that share a similar terminus, known as the angle's vertex, and are referred to as the angle's arms.



Activity:
Ascertain which angles subtend (span) the following: a semicircle, half of a circle, a quarter circle, an eighth of a circle, and a hundredth of a circle.

Dimension
If you were on an item, the number of independent directions you might move in is equal to the object's dimension. A point has zero dimensions because, if you were a prisoner imprisoned in it, you would have nowhere to go. Because you can only move in one direction when walking on a line, a line is one-dimensional. (Forward and backward count as one positive direction and one negative direction in the same direction.) Two independent directions, such as east/west and north/south, are possible to move in an airplane; These two distinct directions combine to form all other directions.

A plane has two dimensions as a result. You can walk autonomously in three directions in a three-dimensional space, like the actual world: up and down, east and west, and north and south. Another way to think about dimensions is the quantity of coordinates needed to find a spot. If a line can identify a point using just one coordinate, such x, then it is said to be one-dimensional. Because it needs two coordinates, like x and y, to find a point, a plane is two-dimensional. Points in three dimensions can only be located using three coordinates, x, y, and z.

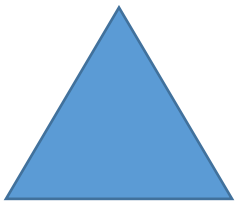
The geometry has been classified into two branches:

1. Plane geometry or 2-dimensional geometry.
2. Space geometry or 3-dimension geometry.

The geometry of two-dimensional objects is known as plane geometry. Here, we look at challenges with the most prevalent two-dimensional objects: circles and polygons. The following geometric shapes make up the plane geometry:

a. **A triangle** is a geometrical shape formed by joining three intersecting line segments with the condition that the sum of all angles is 180°. The point where two line-segments intersect (meet) is called a vertex, so, a triangle has three vertices and three angles. In architecture and landscapes, triangular objects are often used Equilateral triangles are frequently used in construction, as seen at the Pyramid Complex at Giza, Egypt. The four triangular sides of the pyramids are composed of equilateral triangles.

b. Given that the pyramids have stood for more than 4,000 years, these serve as prime illustrations of the triangle's architectural strength. In the East Building of the National Gallery of Art in Washington, D.C., isosceles triangles were used in architectural design. One of the most innovative buildings in the world is the Flatiron Building in New York City. This building's triangular shape comes from the fact that it is situated on a triangular Manhattan block. Triangles are not limited to the core design of a building; they are frequently utilized as embellishments. Triangular windows are frequently found in church-stained glass or as window frames, presumably symbolizing the Holy Trinity. Triangles, both equilateral and isosceles, are employed in the Hearst Tower in Manhattan to provide additional support for the tower and to frame the all-glass window system. In structural engineering and architecture, a truss is a tool for providing structural support. A truss is constructed from a framework made up of numerous tiny triangles.



In literature, we come across six types of triangles:

i. Scalene triangle (triangle in which every side has a different length).

ii. Isosceles triangle (triangle in which any two sides have the same length).

iii. Equilateral triangle (triangle in which every side has the same length).

iv. Right-angled triangle (triangle in which one angle has measurement 90°).

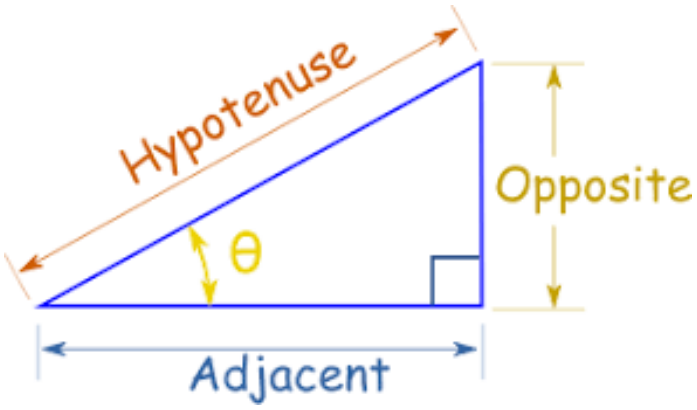
v. Acute angled triangle (triangle in which every angle has measurement less than 90°).

vi. Obtuse angled triangle (triangle in which at most one angle has measurement greater than 90°).

The Pythagorean Theorem (associated with right angled triangle) is a geometric assertion that illustrates the connection between a right triangle's side lengths.

The triangle on the right

equation is $(\text{adjacent})^2 + (\text{opposite})^2 = (\text{hypotenuse})^2$.



The Pythagorean Theorem lets you figure out how long the diagonal that connects two straight lines is. Architectural, woodworking, and other physical construction tasks often employ this application. Building construction also makes use of the Pythagorean Theorem to ensure that structures are square. In two dimensions, the Pythagorean Theorem is helpful for navigation. Surveying can also benefit from it.

A polygon having four sides or edges is called a **quadrilateral**. A quadrilateral's properties are shared by a number of polygons. Only two shapes—rectangles and

squares—have four right angles, despite the fact that at least six other shapes can be classified as quadrilaterals.

Click the link for the use of quadrilaterals in architecture.

http://www.youtube.com/watch?v=_KIBMC8V52g

Literally translated, quadrilateral means "four sides." Any mathematical shape with four edges and four corners can be referred to as it; the edges, however, have to be straight lines. The requirement that the total of the inner angles equal 360 degrees is another characteristic of a quadrilateral. A shape is deemed to be a quadrilateral if it falls inside these boundaries.

- One type of quadrilateral with four right angles is a **rectangle**. A rectangle is defined as a shape with four sides and four right angles. This indicates that a rectangle has 90 degrees at each angle. The opposite sides of a rectangle are also equal in length and parallel to one another.

- The second kind of quadrilateral with four right angles is a **square**. A square is defined as a shape having four equal sides and four right angles. The length of each side of a square is the same, setting it apart from a rectangle. Additionally, opposite sides run parallel to one another.

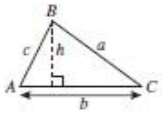
- A quadrilateral's dimensions are met by a number of other shapes, even though they lack four right angles. A quadrilateral having parallel opposite sides is called a parallelogram. A parallelogram does not always require four straight angles, in contrast to a rectangle. A quadrilateral with equal lengths on all four sides is called a rhombus. A rhombus does not always have four straight angles, in contrast to a square. A form with only one pair of parallel, opposite sides is called a trapezoid.

Cubes and cuboids are three-dimensional shapes. A room is shaped like a cube if its walls, floor, and roof are all square (cubical room). A cuboid shape is assumed when the faces are rectangular.

Activity 1. You should be able to identify the quadrilateral you have made based on the information you have obtained. Please complete the following lines to provide an overview of the characteristics of parallelograms that you have learned about. If the property is true, put a checkmark by it, and if it's false, put an X.

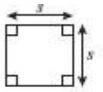
	Opposite sides are congruent	Opposite angles are congruent	Consecutive angles are supplementary	Diagonals are perpendicular	Diagonals are congruent	Diagonals bisect each other.
Parallelograms						
Rectangles						
Squares						
Rhombus						
Isosceles Trapezoids						

Activity-2. Learn the following formulas for the area and volume of different known geometrical shapes.



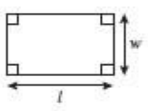
$A = \frac{1}{2}bh$

$A = \frac{1}{2}ab \sin C$



$p = 4s$

$A = s^2$



$p = 2l + 2w$

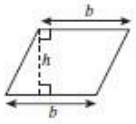
$A = lw$



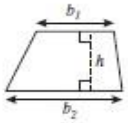
$C = 2\pi r$

$C = \pi d$

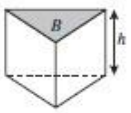
$A = \pi r^2$



$A = bh$



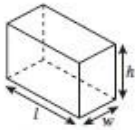
$A = \frac{1}{2}h(b_1 + b_2)$



$V = Bh$

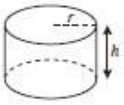
$L.A. = hp$

$S.A. = hp + 2B$



$V = lwh$

$S.A. = 2lw + 2lh + 2wh$



$V = \pi r^2 h$

$L.A. = 2\pi rh$

$S.A. = 2\pi r^2 + 2\pi rh$



$V = \frac{4}{3}\pi r^3$

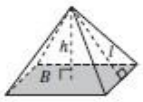
$S.A. = 4\pi r^2$



$V = \frac{1}{3}\pi r^2 h$

$L.A. = \pi rl$

$S.A. = \pi r^2 + \pi rl$



$V = \frac{1}{3}Bh$

$L.A. = \frac{1}{2}lp$

$S.A. = \frac{1}{2}lp + B$

A: for the area of the shape.
P: for the perimeter of the shape.
V: for the volume of the shape.
L.A: for lateral area of the shape.
S.A: for the surface area of the shape.

Example: (City Park Problem).
Two sets of parallel streets surround a city park that is one block in size. The block's streets are all 55 yards long, with a 39-yard perpendicular separation between them. How much sod needs to be bought in order to completely cover the park in grass?

Solution:
The municipal park has a parallelogram-like design, measuring 39 yards in height and 55 yards in base. The parallelogram's area is

$$Area \ = \ b \times \ h \ = \ 55 \, yd \ \times \ 39 \, yd \ = \ 2145 \, yd^2.$$

The city will need to purchase 2145 square yards of sod for the park.

Example: (Building Stairs Problem)
In your recently constructed home, you have a stairway and would like to install plywood over the area underneath the stairs. What is the region's area?

Solution:
The area that needs to be covered is triangular, measuring 12 feet at the base and 9 feet at the top. This triangle's area is

$$Area \ = \frac{1}{2} \times b \times h = \frac{1}{2} \times 12\text{ft} \times 9\text{ft} = 54\text{ft}^2$$

The region that needs to be covered has a 54 square foot area.

Example: (Water Reservoir Problem)
A thirty by forty-meter rectangular foundation and fifteen-meter-tall vertical walls make up a water reservoir. The reservoir was full by the time summer arrived. The water was four meters deep at summer's end. In all, how much water was consumed?

Solution: Because of its rectangular, prism-like shape, the reservoir's volume is calculated by multiplying its length by its breadth by its depth. At the start of summer, when the reservoir was full, the amount of water was

$$30 \, m \ \times \ 40 \, m \ \times \ 15 \, m \ = \ 18{,}000 \, m^3.$$

As summer ended, there was the following quantity of water remaining:

$$30 \ \times \ 40 \ \times \ 4 \, m \ = \ 4800 \, m^3.$$

The amount of water used as a consequence was

$$18{,}000 \, m^3 \ - \ 4800 \, m\text{^}3 \ = \ 13{,}200 \, m^3.$$

Example: Is it possible for one that is 4 inches tall and has a diameter of 3 inches to hold more soup than the other one?

Solution: Remember that 1/2×diameter equals radius. Given that soup cans are shaped as right circular cylinders, the two cans' respective volumes are:

$$\text{Can 1: } V = \pi \times r^2 \times h = \pi \times (1.5\text{in})^2 \times 4 \text{ in} \approx 28.27 \text{ in}^3.$$
$$\text{Can 2: } V = \pi \times r^2 \times h = \pi \times (2 \text{ in})^2 \times 3 \text{ in} \approx 37.70 \text{ in}^3.$$

The second can vehave a bigger volume since it has a larger radius but a lower height.

6.5 Geometry and Architecture

Whether deliberately or accidentally, arithmetic has long been a vital but fundamental instrument utilized by architects, builders, and construction specialists for the spiritual purpose of designing, carrying out, and finishing building projects. Historically, mathematicians have also been architects, and some mathematicians have also been architects. Vitruvius was a highly renowned mathematician and architect. Later, proportions in construction were calculated using Pythagoras' mathematical readings. Leonardo Da Vinci, a well-known worker and proponent of the golden ratio, was also an architect in addition to his many accomplishments.

This chapter's methodology aims to produce conclusions on the value of mathematics in architecture, much as it does in geometry, from crucial site study to the elevation or façade's final design. The entire chapter's goal is to develop mathematical functions associated with architectural engineering and building construction measurement.

6.5.1 History of Use of Geometry in Architecture

In ancient times, men constructed things to suit their wants for space. Royal men acted like royalty by using buildings. In addition to decorative accents, their homes had a natural geometry, partly because of the structural qualities of the materials that were readily accessible in the right diameter. We use the Sumer Reed home, built circa 4000 B.C., as an example. Standard building elements were the robust tall reeds of the Euphrates delta. These were gathered into bundles and fashioned into a pointed or circular arch. The entire house was covered in mud and filled with reed matting. The structural components of the Gothic or Romanesque cathedral styles are all present in the simple, geometric house.

The greatest illustration of Egyptian geometric understanding is found in the pyramids. Pyramids are a reflection of their attempt to represent stability by basing their human world on "cosmic order." There, astronomy and religion had a close relationship with geometry. The "occult geometry system" served as the foundation for all of that era's building. Religious authorities regarded this system of measurements, proportions, and dimensions as holy and heavenly. The basic idea of building design during the Greek Period was the tapering of columns and the manifestation of proportion and visual impacts of buildings according to the dimensions of columns of different orders.

In summary, the Gothic Period's geometric and technical accomplishments were the differentiation of bearing columns from non-bearing walls, the use of pointed arches, the invention of flying buttresses, the use of ribs to support vaults, and an extensive use of glass and tracery in a variety of shapes and patterns. The 13th-century mason used very clever and perceptive trial-and-error, supported by experience and geometric design guidelines, to solve his structural issues rather than using software and analysis as a modern engineer would. The shape was more important to the design than figuring out how much force was acting on it. Similar to this, geometric design was also used in buildings during the Renaissance.

That was when engineering and architectural designs first appeared. An important event in both geometry and architecture was the creation of projective geometry (orthographic and perspective). The Modern idea of building design, on the other hand, was centered on the practicality of using materials like concrete, steel, and glass. This idea is reflected in buildings from the Modernism era and beyond, which cleverly use geometry and shape to maximize every square inch of available space.

6.5.2 Importance of Geometry in Architecture

Geometric space is the foundation of architectural space. Particularly during the design phase, architecture is conceptualized in terms of a geometric space. In his analysis of the connection between geometry and architecture, Robin Evans states that people typically look for geometry in building shapes, followed possibly by the shapes of architectural designs. These are the places where geometry has generally been inactive and stolid. However, the area in between and at either end has seen geometric activity. Evans claims that there is a misinterpretation of the function of geometry throughout the history of building. He discusses the connections between Jean-Nicolas Louis Durand's philosophy of architecture and Gaspard Monge's Descriptive Geometry in his historical study.

About 1800, Durand and Monge shared a classroom at the Ecole Polytechnique in Paris where they taught architecture. Durand created an architecture-specific universal planning grid. Evans explains that a misunderstanding of the spatial coordinate system is the foundation of Durand's grid architecture. Rather than adopting an abstract grasp of the coordinate system, he directly converted the coordinate planes into the floor and walls of architecture.

6.5.3 Geometry for Strength

The most crucial component of constructing a strong structure is the foundation, which is made up of straightforward cubes with a rectangle base that is easy to assemble, maximizes efficiency, and works well with the design. However, pile foundations are made in a cylindrical shape because, when drilled into the earth, minimal friction is needed, and a cylinder with the same volume and less surface area works best. Conversely, the triangle is said to be the most stable shape. For the same reason, they are employed as the most dependable building components for load-bearing and stable buildings, whether they are large trusses or geodesic domes.

6.5.4 Geometry for Performance

Shape selection for maximum output is a simple way to conceptualize geometric applications for performance in architecture. Similar to a dome, a hemispherical roof has many advantages over a flat roof. It requires less room for columns because the weight of the roof is immediately transferred to the hemisphere's rim, and the useless hemispheric space can be used to store hot air that will eventually be released from the roof through a stack effect. Conversely, conical roofs are typically utilized to cover tower-like structures, while pitched triangular prism-shaped roofs are employed when snow and runoff are the primary concerns.

6.5.5 Islam and Architect

Islamic ornamentation heavily employs geometric designs that have evolved over the ages. Numerous of them have roots in ancient Greek, Roman, Byzantine, Central Asian, and Persian cultures. They are typically separated from Islamic art ornamentation based on bending and branching plant patterns, known as the arabesque. However, there are occasions when linear geometric patterns and vegetation are blended into a single design, and some strictly abstract linear patterns take on styles that are obviously inspired by vegetal arabesque patterns. The geometric patterns have developed into exquisite and extremely intricate patterns that are still utilized in many contemporary contexts.

In Islamic architecture, the square and rectangle are important geometric shapes. This is due, in part, to façades constructed with rectangular bricks. In the intense desert sunlight, this ornate brickwork throws shadows that give the impression of three dimensions. A smaller core square inside a larger square that has been rotated 45 degrees is a common theme. Weaved baskets are another source of the square motif.

The principal region for buildings with ornamental brickwork, particularly in the Seljuk period, is the Persianate world; an additional example can be found further west in the Great Mosque of Cordoba. Another popular motif in Islamic architecture is the eight-pointed star, which is frequently seen in tile work and other media. When the outer points are connected and additional intersections form systematic connections, star patterns become incredibly complicated. Repeating motifs can be seen in the tile and stucco décor of the Alhambra palace in Granada, Spain. In Islamic architecture, octagons can take on a variety of forms. They happen on marble floors all the time. The marble opus sectile floors at the Syrian Citadel of Aleppo make use of both the square and the eight-pointed star.

6.5.6 Fractal Geometry

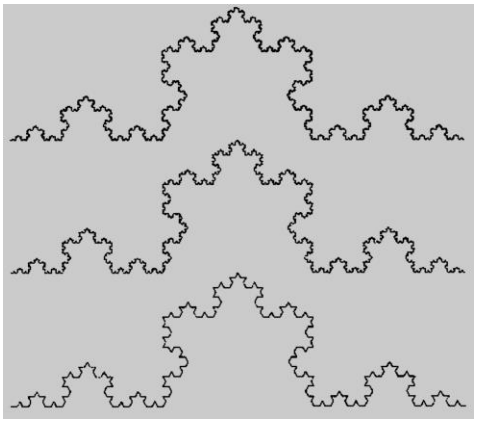
The Weierstrass function, which is continuous everywhere but differentiable nowhere, was introduced by mathematician Karl Weierstrass in 1872, marking the beginning of the mathematical history of fractals. Helge von Koch improved the Weierstrass function's formulation in 1904 and provided a more geometric definition of a related function that is now known as the Koch snowflake. Waclaw

Sielpinski created self-similar patterns and the generating algorithms for them in 1915. Another self-similar fractal example was provided by Georg Cantor. Fractals were explored further in the late 19th and early 20th centuries by Henri Poincare, Felix Klein, Pierre Fatou, and Gaston Julia. When Mandelbrot combined all of this work in 1975, he called it a "fractal."

Limits of iterative schemes using generators of iterative functions on metric spaces can be used to generate fractals. The most widely used, versatile, and potent mathematical tool for creating fractals is the Iterated Function System (IFS). For the iteration process to provide the fractal set, convergence is required. Consequently, rigorous contractions with the Banach fixed-point property are the only functions that can be iterated. The Cantor set, Sierpinski Triangle, Menger sponge, Dragon curve, Space filling curve, and Mandelbrot set are a few of the most outstanding instances of fractal geometry.

Furthermore, IFS offers a link between fractals and real-world pictures. It's a valuable tool for researching fractal collections as well. An introduction to some fundamental fractal set geometry will be discussed in the following from an IFS viewpoint. In a basic scenario, IFS operates on a segment to produce contracted copies of the segment that, in accordance with predetermined guidelines, can be organized in a plane. Complex geometric patterns that repeat endlessly at various scales are known as fractals. They are widely distributed in nature and come in a variety of shapes and sizes. Some natural instances of fractals are as follows:

Snowflakes: Snowflakes, with their distinct six-fold symmetry, display complex fractal patterns. A self-repeating pattern is produced as the branches of growing snow crystals spread out and split into smaller branches.



Trees: The branching structure of trees follows a fractal pattern. Starting from the trunk, branches split into smaller branches, which further split into twigs and leaves. This pattern repeats as you zoom in, creating a self-similar structure.

Mountain Ranges: The jagged edges and peaks of mountain ranges display fractal characteristics. As you zoom in on a mountain, you observe smaller ridges, valleys, and peaks, reminiscent of the larger structure.

Coastlines: Coastal boundaries exhibit fractal properties. Regardless of the scale, as you measure the length of a coastline, it increases due to the intricate shapes of bays, inlets, and smaller features repeating along the coastline.

Clouds: Cloud formations often display fractal patterns. Clouds consist of billows, wisps, and smaller cloud formations that mimic the larger cloud structure, creating a fractal-like appearance.

Ferns: The branching patterns found in ferns exhibit fractal properties. Each leaflet of a fern contains smaller leaflets, and this self-repeating pattern occurs as you examine different sections of the fern.

Lightning: The branching structure of lightning bolts can be considered fractal in nature. The main bolt branches into smaller branches, which further branch into even smaller ones, creating a fractal-like pattern.

Coral Reefs: The intricate structures of coral reefs exhibit fractal properties. When you examine a coral reef, you can observe self-similar patterns at different scales, from the large coral formations to the small individual coral polyps.

Flower Petals: The arrangement of flower petals often follows a fractal pattern known as phyllotaxis. The petals form spirals that occur in a consistent pattern, with each spiral being a reduced-scale repetition of the larger one.

River Networks: River systems exhibit fractal-like patterns as they branch and form tributaries. Regardless of the scale, the river network structure repeats with smaller streams and tributaries mimicking the overall pattern.

These examples demonstrate how fractals are prevalent in the natural world, showcasing the beauty and complexity of nature's design.

6.5.7 Modeling with Geometry

Geometry plays a major role in our comprehension of the three-dimensional world we live in. The concepts used in land surveying for new parks, Earth observation analysis from satellite images, ship navigation at sea, and medical imaging all date back to the ancient Greeks. This chapter demonstrates how geometry, both ancient and modern, aids in our understanding of the surrounding environment.

The Greeks were largely responsible for the development of classical geometry. Although this geometry is still useful to us in many contexts, it is less suitable for forms seen in the natural world. Fractal geometry is a novel kind of geometry that

has arisen in recent decades. It has shown to be so successful at capturing the essence of nature that realistic images are now created with it in both art and movies.

Example: (Measuring the Perimeter of Central Park). Suppose you are asked to determine the circumference of Central Park in New York City, which was designed with a rectangle: https://www.centralpark.com/search/location/central-park-map/map_search.html#lat=40.7824&lng=-73.9643&zoom=14: Assume that your starting point is a 10-meter measuring stick. The circumference of the park may be calculated by multiplying the total number of components by their 10-meter length. For the purposes of this example, let's say you are using a 1-meter ruler, or something smaller. You are only measuring the straight sides of the park, thus the differences in your measures with the 1-meter and 10-meter rulers won't be too great. Put another way, ten 1-meter rulers and a single 10-meter ruler may be used to create a straight line. The length of the ruler won't actually have a significant impact on the size of Central Park.
Example: (The Fractal Border of Spain and Portugal)

Portugal states that the distance between it and Spain is 987 kilometers on its international boundary. The boundary is 1214 kilometers long, according to Spain. On the border's position, nonetheless, the two nations concur. How is it possible?

Solution: The border is formed by a range of natural features, such as mountain ranges and rivers. As a result, it is fractal, with increasing detail being visible upon closer inspection. If the border is measured with a shorter "ruler," it will also be longer, similar to the snowflake curve. Therefore, if Spain and Portugal used separate length "rulers" to measure the border, they could agree on its location but disagree on its length. Spain must have measured with a shorter ruler because it claims a longer border.

6.6 Dealing with Social and Economic Issues Involving Geometrical Objects

Social and economic issues involving geometrical objects can be approached from various perspectives. Here are a few examples of how such issues could be analyzed and addressed:

6.6.1 Urban Planning and Inequality

Geometrical objects like city layouts, zoning, and transportation networks can have a significant impact on social and economic disparities within a community. Analyzing the distribution of resources, services, and amenities within different geometric regions can help identify areas of inequality. Planners can then propose strategies such as equitable zoning policies, improving transportation access, or implementing community development programs to address these issues.

6.6.2 Land Use and Environmental Concerns

Geometrical objects are often used to represent land parcels, and their allocation and usage can affect environmental sustainability. For instance, improper land use and geometrical fragmentation of natural habitats can lead to biodiversity loss, ecosystem degradation, and climate change. Implementing geometrical approaches like green infrastructure planning, conservation corridors, or smart growth policies can help mitigate these issues.

6.6.3 Infrastructure Development and Accessibility

The design and placement of geometric structures such as roads, bridges, and public transportation systems can influence accessibility and social inclusion. Assessing geometric relationships between infrastructure and communities can reveal disparities in access to essential services, education, healthcare, and employment opportunities. By incorporating principles of spatial equity and universal design, urban planners can create more inclusive geometric configurations to address these challenges.

6.6.4 Product Design and Manufacturing Efficiency

Geometrical considerations play a vital role in product design and manufacturing processes. Optimization of geometrical shapes, materials, and assembly techniques can lead to more efficient production systems, reducing costs and environmental impacts. By implementing concepts like lean manufacturing, design for manufacturability, and design for sustainability, companies can address social and economic concerns associated with resource consumption, waste generation, and worker safety.

6.6.5 Trade and Globalization

Geometrical objects can also represent trade networks and supply chains, which have far-reaching social and economic implications. Analyzing the geometry of global trade patterns, logistics networks, and value chains can help identify areas of unequal distribution of resources, labor exploitation, or environmental degradation. Policymakers and businesses can work towards fair trade practices, ethical sourcing, and sustainable supply chain management to promote social equity and responsible economic development. These examples illustrate how geometrical objects can be used as tools for analysis, planning, and decision-making to address social and economic issues. By considering the spatial relationships and configurations of objects, policymakers, researchers, and designers can develop more effective and equitable strategies to tackle these challenges.

Activity-3. (Social activity). For the subdivision, build a play area and play equipment with quadrilaterals and three-dimensional figures. Determine the pricing of the lots and houses using local economics. Create a model house plan. For development to start, the finished product needs to seem like a comprehensive subdivision plan that is prepared for marketing with realtors. In order to find design concepts, safety regulations, and local property values, students will conduct research at several levels.

6.7 Self-Assessment Questions

1. 50 meters long, 30 meters wide, and 2.5 meters deep are the dimensions of a competitive swimming pool. What is the capacity of the pool's water?

2. The floor of an arena is 40 by 50 meters in size, and the roof is 8 meters high. What is its air capacity, expressed in cubic meters? In liters?

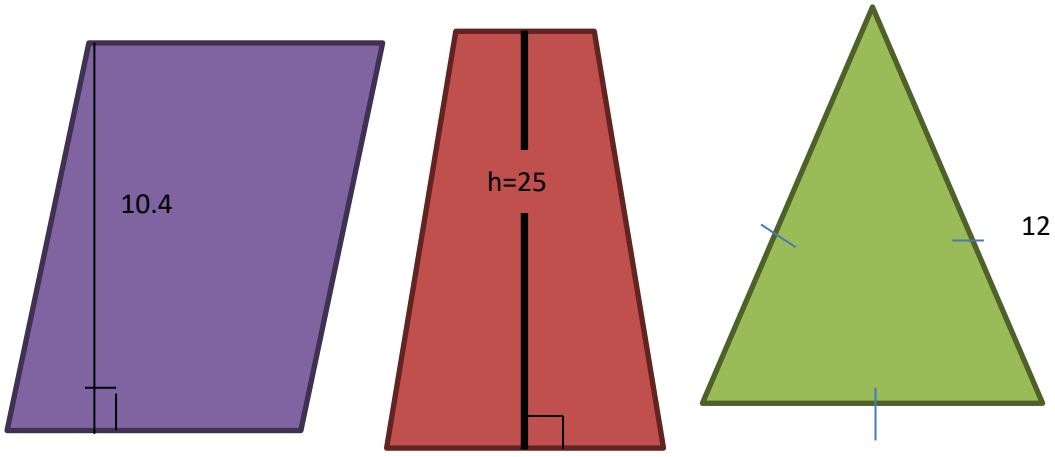
3. The air duct in a stadium measures 40 feet in length and 18 inches in radius on a circular cross-section. What is the volume of the duct and how many square feet of paint are needed to paint the outside of the duct?

4. A grain storage facility is a hemispherical shell with a radius of thirty meters. What is the volume of the building? How much paint is needed to cover the outside of the building completely?

5. A cylindrical container can hold three tennis balls properly piled inside of it. The height of the can or its circumference which is larger? Describe your thinking.

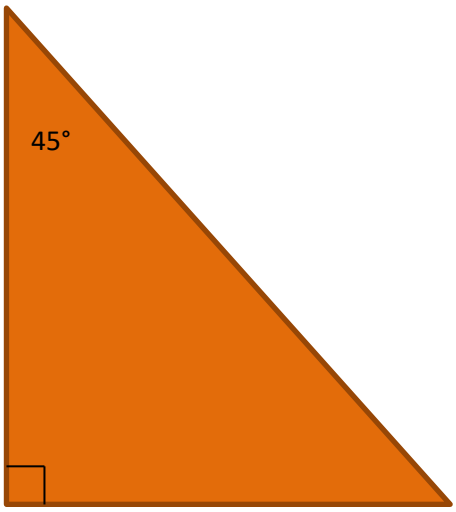
6. Calculate the size and perimeter of:
   i. A square state park with six-mile sides
   ii. A fourteen-inch-wide by eight-inch-long rectangular envelope
   iii. A parallelogram with 8- and 30-foot-long sides and a 4-foot gap between the 30-foot sides

7. Find the area of the following shapes.

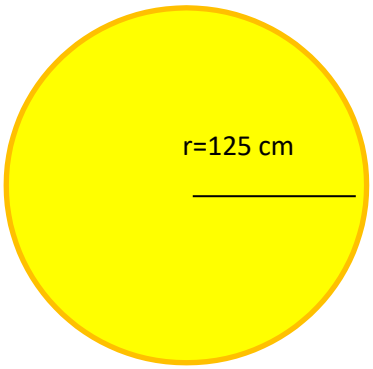


10.4 in 8 m

20.25



32.8cm



Activity-4. Convert the following to amounts of property using the equivalencies stated below:

1 acre = 43,560 square feet.

1 square foot = 0.00002295684 acre.

i. 30 acres = ------- square feet.

ii. 217,800 square feet = ------- acres.

iii. 51 acres ------- square feet.

iv. 136,848 square feet = ---------- acres.

Unit–7

VENN DIAGRAMS

Written by: Dr. Irfan Younas

Review by: Dr. Muhammad Asad Zaigham

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7.1 Introduction

A Venn diagram is a powerful visual tool that illustrates the relationships between different sets. This diagram was invented by John Venn in the 1880s and later named by Clarence Irving Lewis in 1918, in his book “A Survey of Symbolic Logic”. Venn diagrams find applications in elementary set theory, probability, statistics, logic, computer science, and linguistics. They are composed of simple closed curves, typically circles or ovals, enclosed within a rectangle, and are used to represent sets and their relationships.

7.2 Objectives

At the end of the unit (7), you will be able to:

- Understand sets and operations on sets, and their use in daily life activities
- To illustrate operations on sets, use Venn diagrams
- Use Venn diagrams to illustrate facts on economic and social issues
- Deal more confidently with economic and social issues involving Venn diagrams

We already know that propositions are of many kinds. The basic requirement is that a proposition must take one of the clear claims: true or false. Here, we will be focusing on propositions that make claims about the relationship between different categories of things. For instance, when we say, "All natural numbers are integers," It is our contention that the category of integers completely encompasses the category of natural numbers.

There are two common methods for studying these types of propositions. The first method involves using sets, which are essentially collections of objects in mathematics. The second method involves using Venn diagrams, which are visual tools that help to illustrate relationships between sets. Both approaches are crucial for collecting facts and are valuable approaches for enhancing logical thinking skills.

7.3 Relationship Among Sets

A set is a well-defined group of distinct items, which can be living or non-living. For example:

- Set: Officer ranks of Pakistan army
  Members: Field Marshal, General, Lieutenant General, Major General, Brigadier, Colonel, Lieutenant Colonel, Major, Captain, Lieutenant, Second Lieutenant

- Set: Months in an English Calendar
  Members: January, February, March, April, May, June, July, August, September, October, November, December

- Set: Days in a week
  Members: Saturday, Sunday, Monday, Tuesday, Wednesday, Thursday, Friday

Definitions: A set is a group of clearly defined and unique items. These items are referred to as elements of the set.

7.3.1 Set Notation

The following two methods are generally used to describe a set.

- Tabular method or Enumeration method
- Set builder notation or Selector method

1. Tabular method

A common method of writing sets involves placing its members within a pair of braces, {}, and separating each member from the next with a comma. For example, the set days of the week can be written as

$$\{\text{Saturday}, \text{Sunday}, \text{Monday}, \text{Tuesday}, \text{Wednesday}, \text{Thursday}, \text{Friday}\}$$

When there are too many members in a set, it is not practical to list them all. Instead, we use three small dots "..." to show that the sequence of entries proceeds in a similar way. It signifies that the list goes on forever if these little dots show up at the end of the list. For instance, the set of all colors is infinite and can be expressed as ". . .". It is not necessary to list every color; usually, listing three members of the set is enough. If the list ends with three dots, it indicates that the list continues with all other colors.

When a list has three dots in the middle, it means that all other members are included and don't need to be mentioned separately. For example, instead of listing

all lowercase English letters, we can use three dots to represent the remaining letters:

Small English alphabets: $\{a, b, c, \ldots, z\}$

Sometimes, three small dots can be placed at the start of a list. Consider the set of negative integers: $\{\ldots, -3, -2, -1\}$. These dots indicate that the sequence proceeds on the left with smaller integers in consecutive order.

2. Set builder method

In this approach, instead of listing the elements, we describe their characteristics. We specify the qualities that an object must have to be considered an element of the set. For instance;

$A = \{x | \ x$ is a vowel in the English alphabet$\}$

Where the letter x used to describe any random element of the set. The standing line " | " following x is read as 'such that'. We may use ':' to denote 'such that',

Example: Set Identification

Which of the following statements describes a set? Give the reason.

a. A collection of "lovely flowers".
b. A collection of "red flowers".
c. A group of "Young singers".
d. A group of "Players with ages between 18 years and 25 years".

Solution:

a. A group of "lovely flowers" cannot be considered a set because the specific flowers to be included are not clearly defined. The term 'lovely' is subjective, as beauty is in the eye of the beholder. What one person finds appealing may not be to someone else's liking.

b. A set of "Red flowers" consists of all the individual red flowers, making it a well-defined collection with clearly identified objects.

c. The term "Young singers" is ambiguous because it does not specify the age range for what is considered a young singer. As a result, it is unclear which singers should be categorized as young, making the classification of the objects undefined.

d. A group of "players whose ages range from 18 to 25 years" forms a set. This is because the specific age range clearly defines which players are included and which are excluded, making the objects well-defined.

Example: Set
Use the set builder method to rewrite the following sets.

a. Integers greater than -2.

b. The Cold War is generally acknowledged to have started in 1945, following the conclusion of World War II, and to have continued until 1991, when the Soviet Union disintegrated. The Cold War's years in sets.

Solution:
a. The set of integers greater than -2 is $\{-1, 0, 1, 2, 3, \ldots\}$. Three small dots show that the sequence proceeds on the right with greater integers in consecutive order.

b. The years from 1945 to 1991 are included in the list, even if they are not stated specifically. The set of years that comprised the Cold War is {1945,1946, 1947,...,1991}. An indication of its inclusion is provided by the dots.

3. A Brief Review of the Numbers System

In mathematics, we usually discuss a few particular sets of numbers:

The set of natural numbers ($\mathbb{N}$) is $\{1, 2, 3, \ldots\}$, that can be depicted on a real line with equal-spaced points, starting from 1 and extending infinitely to the right.



The set of whole numbers ($\boldsymbol{W}$) is $\{0, 1, 2, 3, \ldots\}$. It can be visualized on a number line, similar to how the set of natural numbers beginning from zero is represented.



All positive and negative whole numbers, including zero, are included in the set of integers (Z). This set of numbers, which extends indefinitely to the left and right, is made up of the following

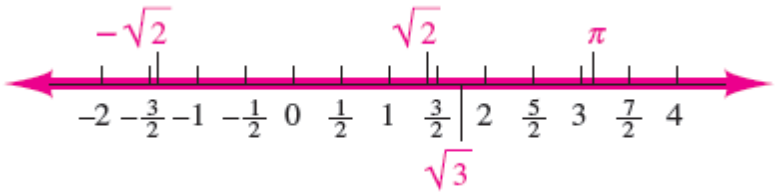
members: {...,-3,-2,-1,0,1,2,3, ……}.



The set of rational numbers ($\mathbb{Q}$) contains all numbers that can be written in the form $\frac{a}{b}$, where a and b are integers and b is not equal to zero. Rational numbers can be further classified into two types by expressing into decimal form: terminating decimal numbers with a finite number of digits in their decimal part (such as 0.20, which is $\frac{1}{5}$), and recurring decimal numbers in which one or more digits repeat indefinitely in their decimal part (such as 0.333 . . ., which is $\frac{1}{3}$).

All the integers that cannot be expressed in the form $\frac{a}{b}$, where a and b are integers, and are included in the set of irrational numbers (Q'). When expressed in decimal form, irrational numbers neither terminate nor are recurring. For example, $\sqrt{2} = 1.414213562\ldots$ is a decimal number which neither terminates nor it is recurring. Therefore $\sqrt{2}$ is an irrational number. Similarly, $\pi = 3.14159265\ldots$ is also an irrational number.

All numbers that fall on the number line, both rational and irrational, are included in the set of real numbers (R). Every point on a number line corresponds to a real number and vice versa. Some selected real numbers are shown on the number line below:



Points to remember:

- Every natural number is also a whole number, an integer, a rational number, and a real number.
- Every whole number is also an integer, a rational number, and a real number.
- Every rational number is also a real number.
- Every irrational number is also a real number.
- Since "Zahlen" is the German term for numbers or integers, the symbol for integers is a "Z."
- The Babylonians, Mayans, and Indians independently invented zero. The number "zero" was first endorsed by an Indian Hindu mathematician. It was not used in any other country until it reached the Indian subcontinent.

7.4 Operations on Sets

1. Union of sets

Let X and Y be two sets. Then union of two sets X and Y denoted by $X \cup Y$ consisting of members which either belong to X or Y (possibly belong to both). Mathematically union of two sets X and Y is defined as $X \cup Y = \{x | x \in X \ or \ \ x \in Y\}$

Note: $X \cup Y$ contains all the elements of X and all the elements of Y with no repetition of elements.

2. Intersection of sets

Let X and Y be two sets. Then the intersection of two sets X and Y denoted by $X \cap Y$ consisting of members which belong to both X and Y. Mathematically intersection of two sets X and Y is defined as $X \cap Y = \{x | x \in X \ \ and \ \ x \in Y\}$

3. Difference of sets

Let X and Y be two sets. The difference between two sets X and Y denoted by $X - Y$ consisting of members that belong to X but do not belong to Y. Mathematically difference of two sets X and Y is defined as $X - Y = \{x | x \in X \ \ and \ \ x \notin Y\}$. The difference $Y - X$ of two sets X and Y can be defined analogously.,

4. Complement of a set

Let X be a subset of U and let U be a universal set. Then complement of X denoted by X^c or X' is actually the difference $U - X$, that is, $X' = U - X = \{x | x \in U \ \ and \ \ x \notin X\}$.

Example: Operations on sets

If $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$, $A = \{1, 2, 3, 4\}$ and $B = \{3, 4, 5, 6\}$, then find

a. $A \cup B$ b. $A \cap B$ c. $B - A$ d. A^c e. B^c

Solution

a. $A \cup B = \{1, 2, 3, 4\} \cup \{3, 4, 5, 6\} = \{1, 2, 3, 4, 5, 6\}$

b. $A \cap B = \{1, 2, 3, 4\} \cap \{3, 4, 5, 6\} = \{3, 4, \}$

c. $A - B = \{1, 2, 3, 4\} - \{3, 4, 5, 6\} = \{1, 2\}$

d. $A^c = U - A = \{1, 2, 3, 4, 5, 6, 7, 8\} - \{1, 2, 3, 4\} = \{5, 6, 7, 8\}$

e. $B^c = U - B = \{1, 2, 3, 4, 5, 6, 7, 8\} - \{\ 3, 4, 5, 6\} = \{1, 2, 7, 8\}$

7.4.1 The Laws of Union and Intersection

a. Commutative law of union
For any two sets A and B, $A \cup B = B \cup A$

b. Commutative law of intersection
For any two sets A and B, $A \cap B = B \cap A$

c. Associative law of union
For any three sets A, B and C, $(A \cup B) \cup C = A \cup (B \cup C)$

d. Associative law of intersection
For any three sets A, B and C, $(A \cap B) \cap C = A \cap (B \cap C)$

e. Distributive law of union over intersection
For any three sets A, B and C, $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

f. Distributive law of intersection over union
For any three sets A, B and C, $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

g. De-Morgan's Laws
For any two sets A and B,
i. $(A \cup B)^c = A^c \cap B^c$
ii. $(A \cap B)^c = A^c \cup B^c$

7.4.2 Understanding the Relationship of Sets by Venn Diagrams

Generally, in a Venn diagram, a rectangle is used to represent a universal set ***U*** and ovals/circles inside a rectangle are used to represent the subsets of ***U***. Consider the following examples to develop better learning about Venn diagrams.

Think about the real and natural number sets. The set of natural numbers contains the set of real numbers because every natural number is a real number. We can show this represent this relation in a Venn diagram with an oval to represent the set of natural numbers inside a rectangle representing the set of real numbers (Figure 7.1). Observe that the diagram explains the relation between the two sets.

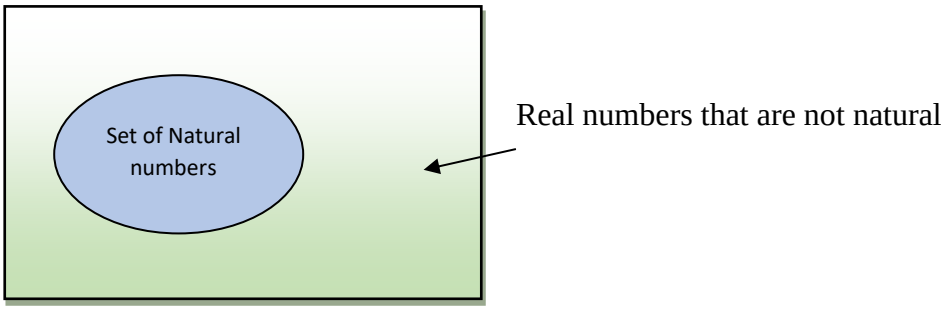


Figure 7.1 Relationship between the natural number set and the real number set

An oval enclosed in a rectangle, created two regions in the diagram: inside the oval represents the set of natural numbers, while the area outside the oval represents the set of real numbers that are not natural numbers, such as terminating and non-terminating, non-recurring decimals.

Next, consider the sets of cows and goats. A livestock animal can be either a cow or a goat, but not both. Therefore, the set of cows and the set of goats are disjoint (Figure 7. 2). Again, we enclose the ovals in a rectangle that represents the all-livestock animals. Here, the area outside the two ovals depicts the livestock animals that are different from cows or goats such as camels, buffalos etc.

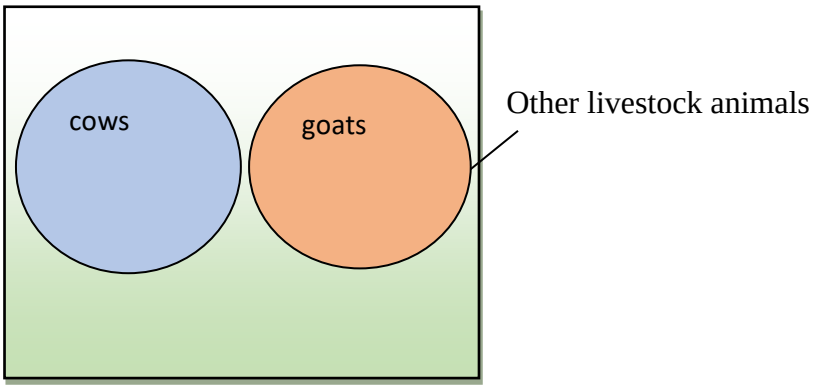


Figure 7.2 The Sets of Cows and Goats are Disjoint.

In the Venn diagram (Figure 7. 3), the sets of lecturers and ladies overlap since a person can be both. The diagram has four regions: the overlapping region for lady lecturers, non-overlapping regions for ladies who are not lecturers and lecturers who are not ladies, and the region outside both ovals for persons who are neither lecturers nor ladies.

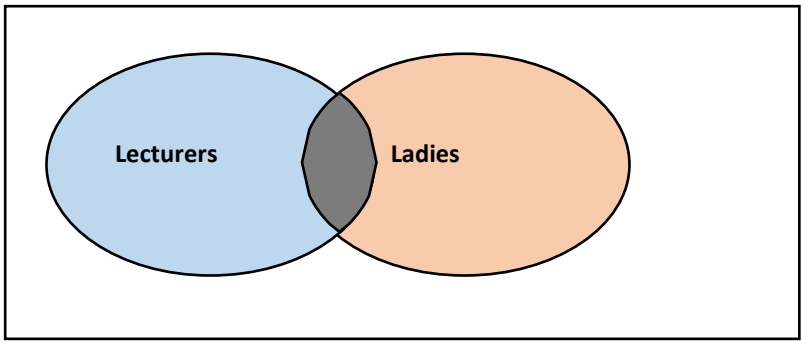
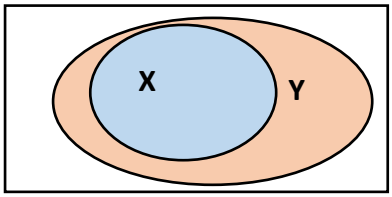
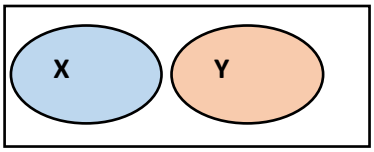
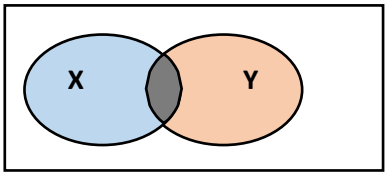


Figure 7.3 The Sets of Lecturers and Ladies are Overlapping.

7.4.3 Set Relationships and Venn Diagrams

Let X and Y be two sets, then they may be associated in one of the following ways:

➢ The Venn diagram may contain an oval **X** completely inside another oval **Y**. In this case, X is a subset of Y means that all entries of X are also in Y.	
➢ The Venn diagram may consist of two non-overlapping ovals X and Y. In this case, X is disjoint from Y means that there is no point in common.	
➢ The Venn diagram may consist of two overlapping ovals of X and Y. In this case, X and Y are overlapping sets means that they have some points in common.	

7.4.4 Venn Diagrams

Explain the relationship between the given sets of pairings, and then draw a Venn diagram to show how the various sections of the diagram are connected.

a) Field Prize winners and Noble Prize winners.

b) People Party workers and Muslim League workers.

Solution:

a) Any person can win both a Nobel Prize and a Fields Medal. Therefore, the overlapping area represents the persons who won both prizes. The Venn diagram (Figure 7.4) illustrates this concept. The area situated outside the two circles signifies individuals who have not achieved either award.

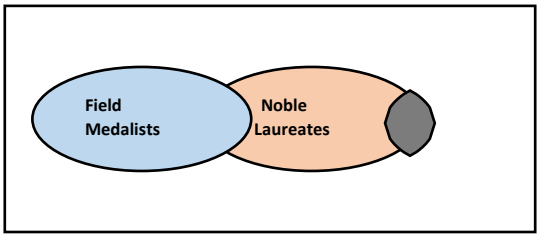


Figure 7.4 The set of Field Medalists and Noble Laureates is Overlapping

b) These sets are disjoint because a person can only be listed with one political party. A Venn diagram is used to show this in Figure 7.5. Those who are registered with other independent or non-registered parties, or those who are not registered with either party, are represented by the region between the two circles.

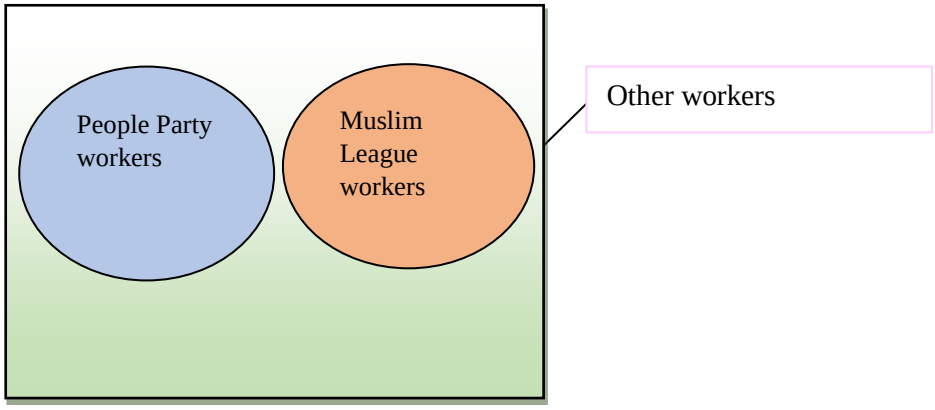


Figure 7.5 The Sets of People Party and Muslim League Workers are Disjoint.

7.4.5 Number Systems

Make a Venn diagram that shows how the whole, rational, real, integer, natural, and rational numbers relate to each other. Where in the graphic are the irrational numbers located?

Solution:

It is important to note that all the natural numbers are whole numbers. Integers are other kinds of whole numbers that contain zero as well as positive and negative integers. Real numbers are made up of all rational numbers as well as irrational numbers that cannot be expressed as fractions, while rational numbers only comprise integers and fractions. To further understand this idea, look at Figure 7.5, which shows a set of circles with an area representing irrational numbers inside the real numbers and outside the rational numbers.

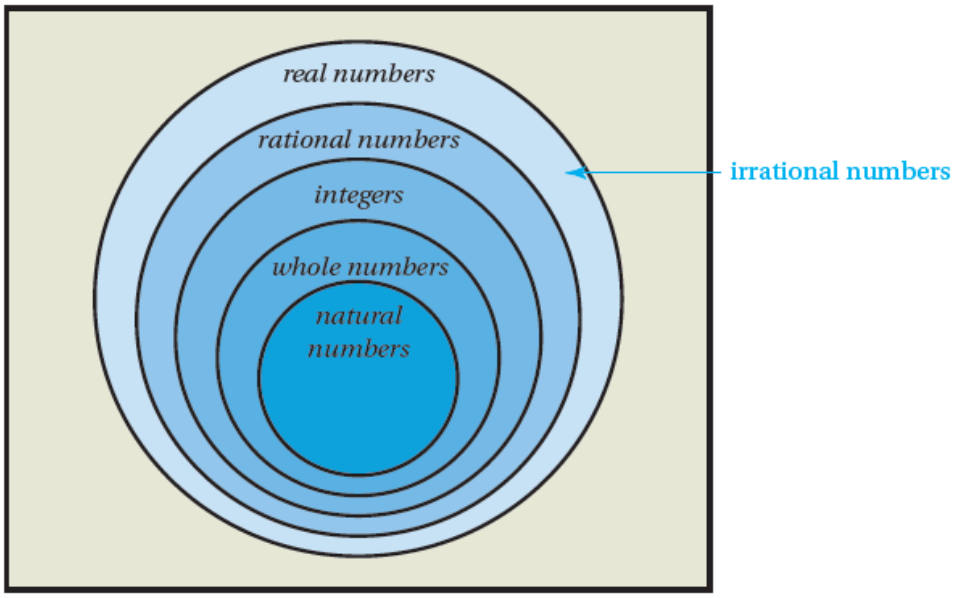


Figure 7.6 Venn Diagram

7.4.6 Blood Types

Depending on whether three antigens—A, B, and Rh—are present or absent, humans have distinct blood types. Antigens A and B are used to determine blood type. Blood is categorized as type A if it only contains antigen A and type B if it only contains antigen B. Blood is classified as type AB if both antigens are present, and type O if none are. The term positive (present) or negative (absent) indicates whether Rh is present or absent. The proportion of each blood type in the Pakistani population is shown in Table 7.1. Create a Venn diagram using this data.

Table 7.1 Blood Types in Pakistani Population

Blood Type	Percentage of Population
A positive	22%
B Positive	30%
AB positive	9%
O positive	30%
A negative	2%
B negative	3%
AB negative	1%
O negative	3%

Solution:
To understand the three antigens, we can divide them into three categories: A, B, and Rh (positive). The various regions can be visualized in the Venn diagram (Figure 7.6), This shows the eight zones, each labeled with the population's kind and proportion. For instance, the middle region, shown by 9%, denotes the presence of all three antigens (AB positive). To make sure that all eight regions are labeled correctly, you should refer to the data in Table 7.1.

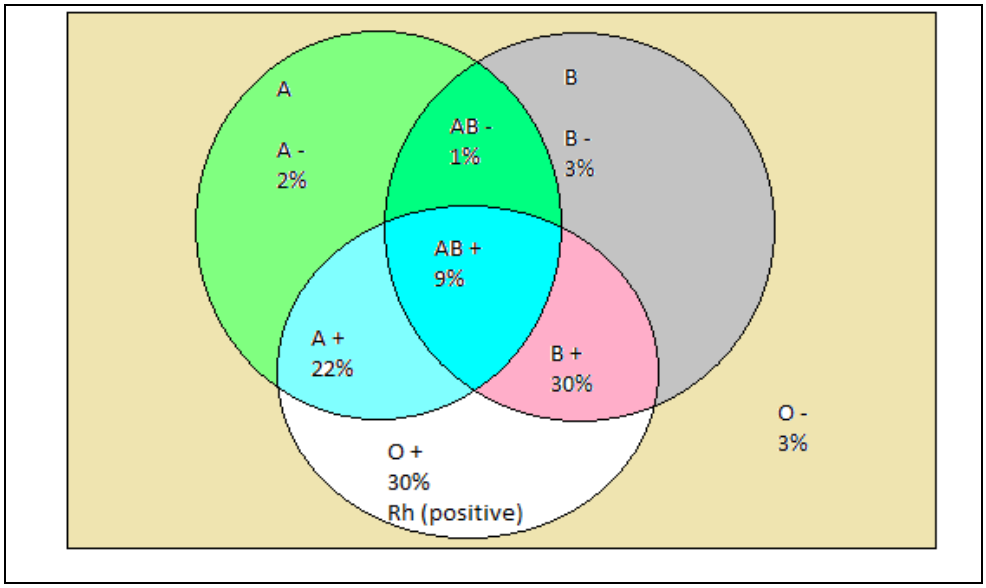


Figure 7.7 Venn Diagram

7.4.7 Colour Combinations
The RGB color model's primary application is image sensing, representation, and display in electronic systems; however, it has also been employed in traditional photography. When joined in pairs, these colors produce the following results, with each color having equal power.

Combination	Result
Green-Red	Yellow
Blue-Red	Purple
Green-Blue	Light blue

To make white, you need to mix red, green, and blue, while black is not created by any of these colors. Create a Venn diagram representing this color information.

Solution:

Figure 7.7 is a Venn diagram that shows the blending of three distinct colors. The colors red, green, and blue are displayed in separate regions without any overlap. In contrast, the colors that result from the blending of two colors, such as red-green, red-blue, and blue-green, are shown in the regions where the circles overlap The centre area displays white, which is the result of the combination of all three colors. Conversely, black appears outside of the three circles, representing the complete absence of any color. By adjusting the strengths of each color, a monitor can create a broader spectrum of colors.

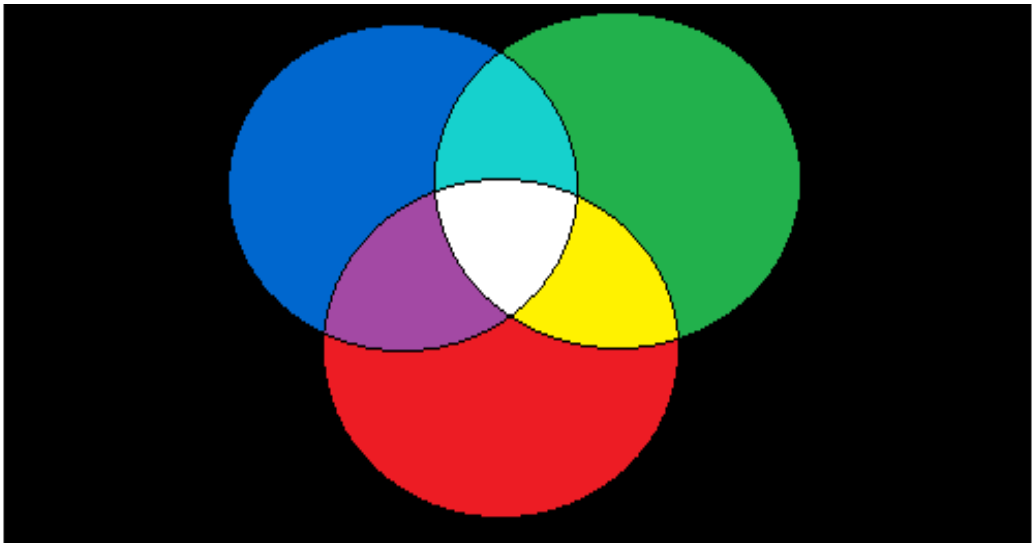


Figure 7.8 Venn Diagram

7.5 Self-Assessment Questions

1. Choose the best option a/b/c/d. Justify your answer briefly.

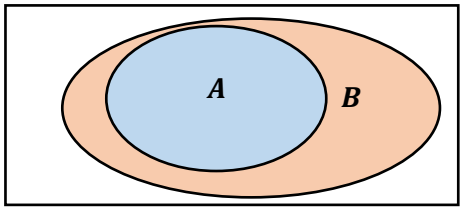
(i) Consider the set {Anhui, Fujian, Gansu, . . . , Zhejiang}. The " . . . " represents

a) that we are not sure about other members of the set.
b) the other 19 provinces of China.
c) Hebei, Henan, and Jiangsu.

(ii) Which of the following does not belong to the set of integers?

a) 0	b) $-\frac{1}{2}$	c) 1

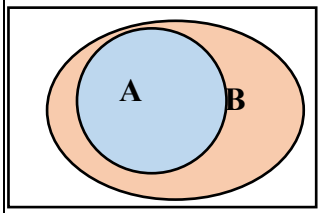
(iii) The relation explaining the given Venn diagram correctly is _____.



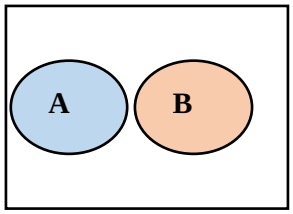
a) ***A*** and ***B*** are disjoint sets.
b) ***B*** is contained in ***A***.
c) ***A*** is contained in ***B***.

(iv) Let ***A*** be the set of all cricketers representing their countries in the Cricket World Cup 2023 and ***B*** be the set of all Pakistani cricketers in the World Cup 2023. Then the Venn diagram explaining the relationship correctly is

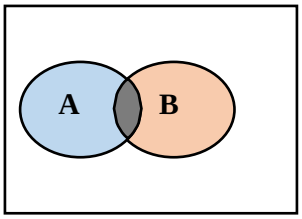
a)



b)

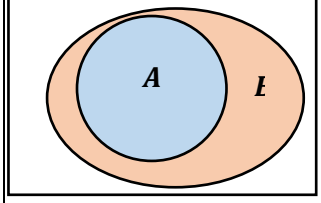


c)

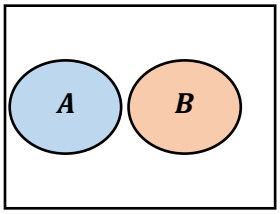


(v) Let B be the set of all Pakistani lawyers and A be the set of all Pakistani female lawyers. Then the Venn diagram explaining the relationship correctly is:

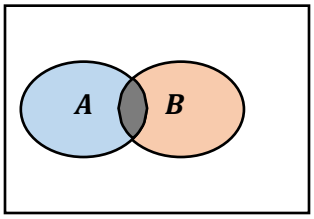
a)



b)

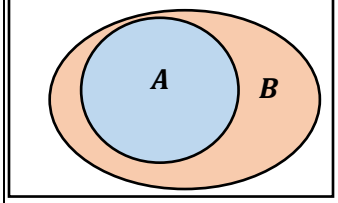


c)

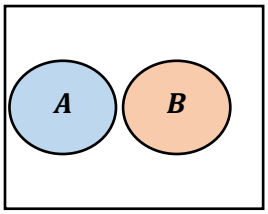


(vi) Let ***A*** be the set of all kinds of mangoes and ***B*** be the set of all kinds of apples. Then the Venn diagram explaining the relationship correctly is:

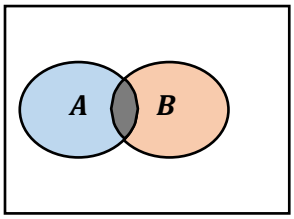
a)



b)

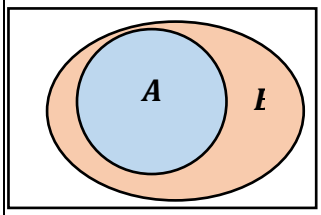


c)

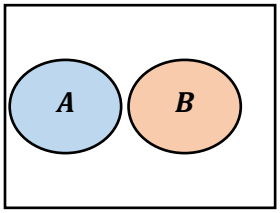


(vii) Let ***B*** be the set of all doctors in the Ganga Ram Hospital, Lahore Pakistan and ***A*** be the set of all female doctors in the Ganga Ram Hospital, Lahore Pakistan. Then the Venn diagram explaining the relationship correctly is

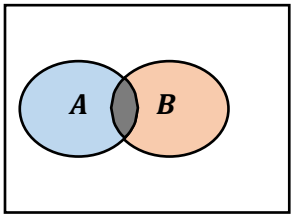
a)

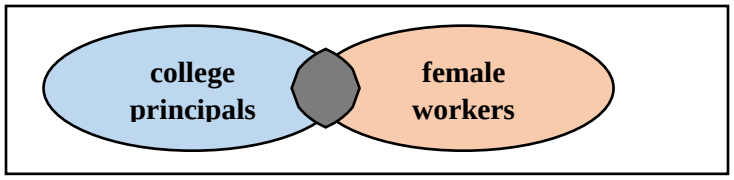


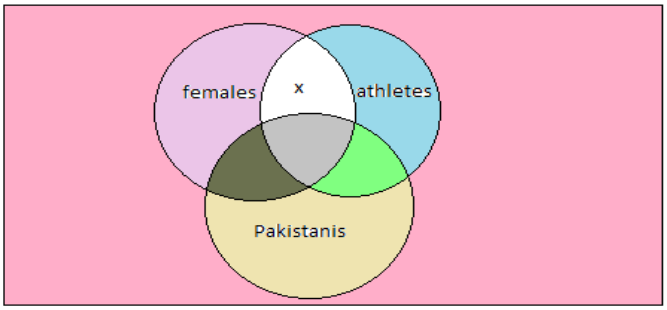
b)



c)



(viii)	The common part in the Venn diagram tells us about  a) Some female workers are college principals. b) No female workers are college principals. c) Some female workers are not college principals.

(ix)	Region X in the Venn diagram given below tells us about  a) Female athletes who are not Pakistanis. b) Female athletes who are Pakistanis. c) Pakistanis' athletes who are not females.

(x)	People who are at the center of the Venn diagram in exercise (x) are represented by a) Pakistani women and athletics. b) Female athletes or Pakistanis. c) Neither a man nor an athlete from Pakistan.

2. What is a set and how do we list its members using braces?

3. Define the Venn diagram and use it to show the relationships of subsets, disjoint sets, and overlapping sets.

4. Describe the kinds of data that may be found in a Venn diagram and provide directions for creating one with three intersecting sets.

Please determine if the statements below are true or false and provide an explanation for your answer.

5. The payments we make on behalf of the IESCO bill are a subset of the payments we make on behalf of the SNGPL Bill.

6. I numbered the pages in my textbook in an unreasonable amount.

7. I went to a school to inquire about the children's breakfast consumption. Next, I use a Venn diagram to explain the findings, where one circle is fully contained inside another (inside a rectangle).

From the sets of natural numbers, whole numbers, integers, rational numbers, and real numbers, select the set that best describes the given values.

8. 45	9. -23	10. $\frac{4}{5}$
11. 2.1375	12. $-\frac{7}{52}$	13. 1296
14. π	15. e	16. 17
17. $\frac{\pi}{3}$	18. -174.354	19. $2e$

Write the members of these sets using set notation (braces) or indicate if the set is empty. Use "..." for patterns.

20. The days of the week

21. The odd numbers between, but not including 3 and 49

22. The countries that share a border with Pakistan

23. The perfect squares between 3 and 65

24. The vowels of the English alphabet

25. Even numbers between 5 and 70 that are divisible by 5

Sketch and explain the Venn diagrams for the given pairs of sets.

26. women and attorneys

27. nurses and paragliders

28. rational numbers and irrational numbers

29. athletes and professors

Sketch a Venn diagram in each statement and label the regions of the diagram. Using the Venn diagram, answer the question followed by each statement.

30. If all widows are women, is it valid to conclude that some women are not widows?

31. Given the premise that no reptiles are mammals, is it feasible that some mammals are reptiles?

32. Taking into account that every member of the Pakistan National Assembly is 35 years old. Is it reasonable to conclude that no one under thirty has ever served as a member of the National Assembly? With the assumption that every child can sing, is it reasonable to conclude that some singers are children?

33. Assuming that no insects are fish, is it feasible that some fish are insects?

34. If plumbers do not engage in deceitful practices, is it plausible that at least one plumber engages in cheating behavior?

35. Is it accurate to assert that no individuals displaying frowns are winners, given the premise that winners smile?

36. If some movie stars are redheads, can the deduction be made that there exist movie stars who are blond?

Draw the Venn diagrams that show the eight regions that each describe the three sets that are provided. Describe each region's members or state if a region is empty.

37. kindergarten teachers, dentists, and women

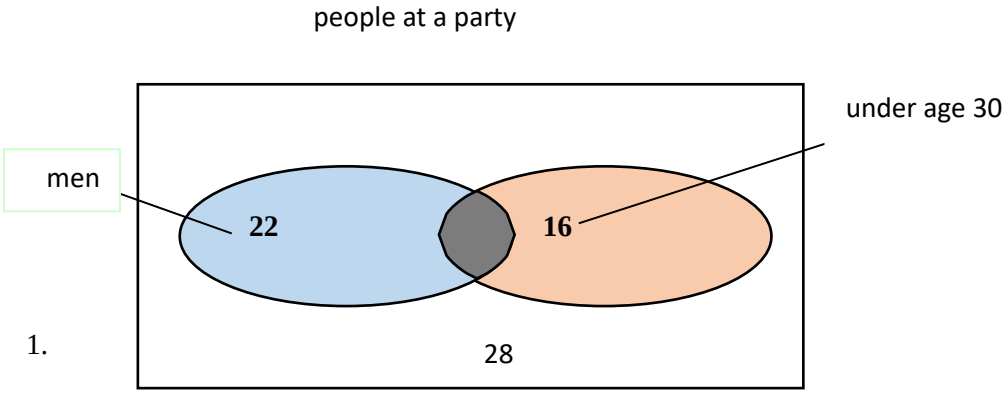
38. hockey players, figure skaters, and women

39. published works, novels, and songs

40. words that begin with *s*, words with less than four letters, and verbs

41. teachers, swimmers, and tall people

In problems 42-43: Use the Venn diagram to answer the following questions.



1.

42.

i. Count under 30 women attending the party.

ii. Count over 30 men attending the party.

iii. Count all women attending the party.

iv. Count all the people attending the party.

43.

i. Count under 30 men attending the party.

ii. Count over 30 women attending the party.

iii. Count all men attending the party.

iv. Count the people attending the party who are not under 30.

44. The table displays the votes (in millions) for the two candidates in Pakistan's 2024 National Assembly elections. Draw a Venn diagram to represent the data.

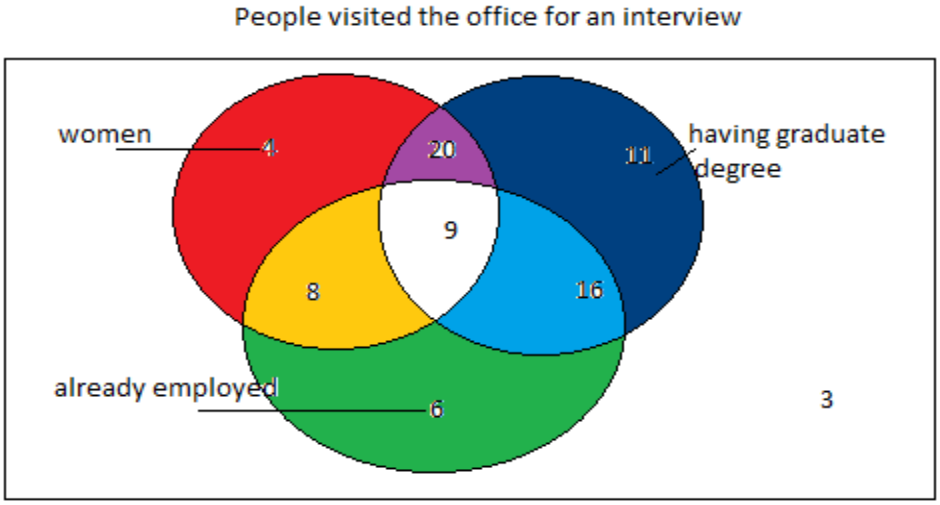
	Imran Khan	Bilawal Bhutto
Women voters	87.45	44.23
Men voters	250.64	102.27

45. Research conducted by Rawalpindi Medical College has studied the impact of tomatoes on cancer. The findings revealed that individuals who consume a high number of tomatoes and tomato products have a significantly lower risk of developing several types of cancers, although not all. To give an illustration, the following table shows the incidence of oral cancer in two different groups of individuals: one group that ate an average of one tomato per day, and another group that ingested fewer than three tomatoes per week. Create a Venn diagram to illustrate the provided information.

46.

	No Oral Cancer	Oral Cancer
One tomato daily	191	9
Less than three tomatoes per week	164	16

In problems 47-48, Answer the following questions using the Venn diagram given below.



47.

i. How many unemployed women candidates visited the office for an interview who have a graduate degree?

ii. How many already employed men visited the office for an interview?

iii. How many already employed women without a graduate degree visited the office for an interview?

iv. How many men visited the office for an interview?

48.

i. How many already employed men without a graduate degree visited the office for an interview?

ii. How many unemployed women visited the office for an interview?

iii. How many unemployed men without a graduate degree visited the office for an interview?

iv. How many people visited the office for an interview?

49. **Drug Use in Hospitals.** Individuals who visited an Islamabad hospital on a single day were prescribed the following amounts of antibiotics (A), painkillers (P), and blood pressure medications (BP):

P only 22	BP and A only 15	BP only 8	P and A only 24
A only 12	P and BP only 16	All three 2	None 2

(i) Sketch the Venn diagram with three circles to summarize the table results.

(ii) How many patients took BP medicines or antibiotics?

(iii) How many patients specifically took BP medicines but did not take any pain medicine?

(iv) What proportion of patients used painkillers at all?

(v) How many patients used antibiotics and blood pressure medications but not painkillers?

(vi) How many patients used pain relievers, antibiotics, or blood pressure medication?

50. **Readership Survey.** A soap manufacturing company decided to spend 130 million rupees on TV/radio, internet, and newspaper advertisements. The details of the amount in millions spent on these advertisements are given below:

TV/radio only	Rs. 20	TV/radio and internet only	12
Internet only	29	TV/radio and newspapers only	18
Newspapers only	15	Internet and newspapers only have	22
None	6	All three sources	8

i. Create a Venn diagram with three circles to summarize the table results.

ii. How many million rupees are spent on radio/TV or the Internet?

iii. How many million rupees are spent on radio/TV or the Internet but not newspapers?

iv. How many million rupees are spent on the Internet but not on TV/radio?

v. How many million rupees are spent on TV/radio but not on newspapers?

51. **Use Venn diagrams to verify the properties of union and intersection using any three sets U, A, B and C.**

i. Commutative property of the union

$$A \cup B = B \cup A$$

ii. Commutative Property of intersection

$$A \cap B = B \cap A$$

iii. Associative property of union

$$(A \cup B) \cup C = A \cup (B \cup C)$$

iv. Associative property of intersection

$$(A \cap B) \cap C = A \cap (B \cap C)$$

v. Distributive property of union over the intersection

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

vi. Distributive property of intersection over union

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

vii. De-Morgan's Laws

i. $(A \cup B)^c = A^c \cap B^c$

ii. $(A \cap B)^c = A^c \cup B^c$

Unit–8

INEQUALITIES

Written by: Dr. Irfan Younas

Reviewed by: Dr. Muhammad Asad Zaigham

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8.1 Introduction

In our everyday experiences, we often encounter situations that can be represented by algebraic inequalities rather than algebraic equations. An inequality resembles an equation, but instead of an equal sign, it contains symbols such as < (less than), > (greater than), $\leq$ (less than or equal), or $\geq$ (greater than or equal). Unlike an equation, inequality typically has an infinite number of solutions. This means that the solution to an inequality forms an interval or a combination of intervals on the real number line when dealing with a single variable.

8.2 Objectives

Exploring inequalities in our surroundings and addressing practical problems involving inequalities in various fields.

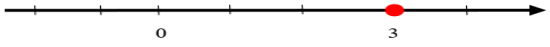
At the end of the unit (8), the students will be able to:

- Differentiate between algebraic expressions, algebraic equations, and algebraic inequalities
- Solve algebraic inequalities
- Understand the inequalities around us
- Design the mathematical model of the linear equalities by given conditions
- Solving practical problems involving inequalities across various disciplines

8.3 Inequalities

In our daily life, many problems lead to algebraic inequalities instead of algebraic equations. It looks like an equation, except that it contains one of the symbols $<, >, \leq, or \geq$ in place of the equal sign. An inequality differs from an equation in that it typically has numerous solutions, resulting in the solution forming an interval or a combination of intervals on the real number line when a single variable is involved. Consider an equation to illustrate the difference between an inequality from its corresponding equation.

u	$6u + 3 \leq 21$
-1	$-3 \leq 21$ True
0	$3 \leq 21$ True
1	$9 \leq 21$ True
2	$15 \leq 21$ True
3	$21 \leq 21$ True
4	$27 \leq 21$ False
5	$33 \leq 21$ False
6	$39 \leq 21$ False
7	$45 \leq 21$ False

	Solution	Graph
Equation: $6u + 3 = 21$	$u = 3$	
Inequality: $6u + 3 \leq 21$	$u \leq 3$	

8.3.1 Properties of Inequality

a. $u \leq v \Leftrightarrow u + w \leq v + w$
This property allows us to add the same quantity to both sides of an inequality to get an equivalent inequality.

b. $u \leq v \Leftrightarrow u - w \leq v - w$
This property allows us to subtract the same quantity from both sides of an inequality to get an equivalent inequality.

c. i. If $w > 0$, then $u \leq v \Leftrightarrow uw \leq vw$
This property tells us that multiplying a positive quantity on both sides of an inequality yields an equivalent inequality.

ii. If $w < 0$, then $u \leq v \Leftrightarrow uw \geq vw$
This property tells us that multiplying a negative quantity on both sides of an inequality reverses the sign of inequality.

d. If $u > 0$ and $v > 0$, then $u \leq v \Leftrightarrow \frac{1}{U} \geq \frac{1}{V}$
This property allows us to take reciprocal on both sides of an inequality involving positive quantities reverses the sign of inequality.

e. If $u \leq v$ and $w \leq x$, then $u + w \leq v + x$
This property allows us that inequalities can be added.

f. If $u \leq v$ and $v \leq w$, then $u \leq w$
This property allows us that transitivity holds in the inequalities.

This property allows us to multiply the non-zero quantity on both sides of an equation to yield an equivalent equation.

How to solve Linear Inequalities?
When an inequality contains terms that are constants or multiples of the variable raised to the 1st power, it is known as a linear inequality. To solve a linear inequality, we group the variable terms on one side of the inequality sign and all the constant terms on the other side.
Example: Solve the linear inequality $2u < 7u + 10$, and represent the solution set on the real line.

Solution:

$$2u < 7u + 10$$
$$2u - 7u < 7u + 10 - 7u$$
$$-5u < 10$$
$$\left(-\frac{1}{5}\right)(-5u) > \left(-\frac{1}{5}\right)(10)$$
Multiply by $\frac{-1}{5}$ $u > -2$ Simplify

The Solution set is $\{u|u > -2\}$, that is, the interval $(-2, \infty)$. In other words the solution set of given inequality contains all real numbers greater than -2. It is graphed in Figure 1.



Figure 1

Example: Solve a pair of Simultaneous Inequalities.

Solve the inequalities $3 \leq 4u - 5 < 15$.

Solution: The solution set of such inequalities consists of all those values of u for which both the inequalities $3 \leq 4u - 5$ and $4u - 5 < 15$ are true. We observe that the inequalities are equivalent to rules 1 and 3:

$3 \leq 4u - 5 < 15$	Given inequality
$8 \leq 4u < 20$	Add 5
$2 \leq u < 5$	Divide by 4

Therefore the solution set is $(2, 5)$, as represented by Figure 2.



Figure 2

How to solve the Nonlinear Inequalities?
When addressing nonlinear inequalities, the method of factoring is applied according to the following principle:

The Sign of A Product or Quotient
The outcome is positive when multiplying or dividing an even number of negative values. On the other hand, the outcome is negative if an odd number of negative values are being multiplied or divided.

Guidelines for Solving Nonlinear Inequalities

1. Adjust the inequality if required such that every term that is not zero is on one side of the inequality sign. Make sure the fractions on the non-zero side have a common denominator if they do.
2. We must factor in the non-zero side of the inequality and determine the values that bring each component to zero in order to solve it. We can create sub-intervals out of the actual number line with the aid of these values. We may then make a list of these intervals.
3. Create a thorough table or graphic that illustrates the indications of each factor at various intervals using particular values. Next, ascertain the sign of the total product or quotient of these variables in the table's last row.
4. Utilizing the attached sign table, determine the intervals when the inequality is true. Next, and especially if the inequality comprises ≤ or ≥, it is crucial to verify if the values at the ends of these intervals fulfill the inequality.

Example: Solve the inequality $u^2 \leq 3u + 4$.

Solution: Given inequality is $u^2 \leq 3u + 4$

$u^2 - 3u - 4 \leq 0$ subtract $3u$ and 4 to move all terms on one side

$(u - 4)(u + 1) \leq 0$ find factors

The factors $u - 4$ and $u + 1$ are zero, when $u = 4$ and -1, respectively. These numbers divide the real line into sub-intervals $(-\infty, -1), (-1, 4), (4, \infty)$ as shown in the **Figure 3**.

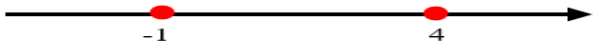


Figure 3

The factors $u + 1$ and $u - 4$ change sign only at -1 and 4, respectively. So, these factors maintain their sign on each of these three intervals.

Interval	Test Value	Result $(u - 4)(u + 1) \leq 0$
$(-\infty, -1)$	$u = -2$ $u - 4$: $-$ ve $u + 1$: $-$ ve	+ve : False
$(-1, 4)$	$u = 0$ $u - 4$: $-$ ve $u + 1$: $+$ ve	$-$ve: True
$(4, \infty)$	$u = 5$ $u - 4$: $+$ ve $u + 1$: $+$ ve	+ve: False

We notice that the inequality is negative on the interval $(-1, 4)$ and the end points -1 and 4 satisfy the inequality. Therefore, the solution set is $\{u | -1 \leq u \leq 4\} = [-1, 4]$ as shown in the **Figure 4**.



Figure 4

Example: Solve the inequality $\frac{1+u}{1-u} \geq 1$.

Solution: Given inequality is $\frac{1+u}{1-u} \geq 1$.

$\frac{1+u}{1-u} - 1 \geq 0$ subtract 1 to move all terms on one side.

$\frac{1+u}{1-u} - \frac{1-u}{1-u} \geq 0$ makes common denominators which is $1-u$.

$\frac{(1+u)-(1-u)}{1-u} \geq 0$ combine the fractions

$\frac{2u}{1-u} \geq 0$ simplify

The factors $2u$ and $1-u$ on the LHS are zero, u is 0 and 1. These numbers divide the real line into sub-intervals $(-\infty, 0), (0, 1), (1, \infty)$ as shown in **Figure 5**.

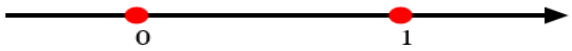


Figure 5

The factors $2u$ and $1-u$ change sign only at 0 and 1, respectively. So, these factors maintain their sign on each of these three intervals.

Interval	Test Value	Result $\frac{2u}{1-u} \geq 0$
$(-\infty, 0)$	$u = -1$ $2u$: $-$ ve $1-u$: $+$ ve	$-$ve : False
$(0, 1)$	$u = 0.5$ $2u$: $+$ ve $1-u$: $+$ ve	$+$ve: True
$(1, \infty)$	u =2 $2u$: $+$ ve $1-u$: $-$ ve	$-$ve: False

We notice that the inequality is positive on the interval $(0, 1)$ and the end point 0 satisfies the inequality but 1 does not satisfy the inequality. Therefore, the solution set is $\{u | 0 \leq u < 1\} = [0, 1)$ as shown in the **Figure 6**.

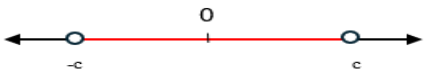
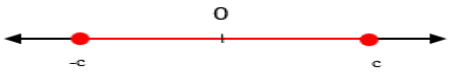
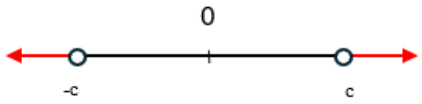
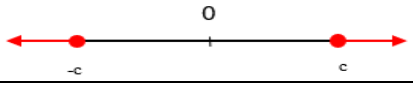


Figure 6

8.4 Absolute Value Inequalities

The following properties have a key role in solving absolute value inequalities.

Properties of Absolute Value Inequalities

Sr. #	Inequality	Equivalent form	Graph
1.	$\|u\| < c$	$-c < u < c$	
2.	$\|u\| \leq c$	$-c \leq u \leq c$	
3.	$\|u\| > c$	$u < -c$ or $c < u$	
4.	$\|u\| \geq c$	$u \leq -c$ or $c \leq u$	

Example: Solve the Inequality $|u - 3| < 2$.

Solution: The given inequality $|u - 3| < 2$ is equivalent to

$-2 < u - 3 < 2$ By property 1

$1 < u < 5$ Add 3

The solution set is $(1, 5)$.



Figure 7

Example: Solve the inequality $|3u + 4| \geq 6$.

Solution: The given inequality $|3u + 4| \geq 6$ is equivalent to

$3u + 4 \geq 6$ or $3u + 4 \leq -6$

$3u \geq 2$ or $3u \leq -10$

$u \geq \frac{2}{3}$ or $u \leq \frac{-10}{3}$

So, the solution set is $\left\{u | \, u \leq \frac{-10}{3} \; or \; u \geq \frac{2}{3}\right\} = \left]-\infty, \frac{-10}{3}\right] \cup \left[\frac{2}{3}, \infty\right[$.

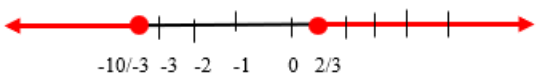


Figure 8

8.5 Modeling of Inequalities

In our day-to-day activities, we frequently encounter situations where we strive to understand and compare different quantities to determine their relative magnitudes. This is particularly important when we aim to establish which quantity is larger or smaller than another.

Example: There are two plans for tickets in a sports gala.

Plan A: Rs. 50 entrance fee and Rs. 25 each attempt in any sports activity
Plan B: Rs. 30 entrance fee and Rs.45 each attempt in any sports activity

How many attempts a man would take for plan A to be cheaper than plan B?

Solution: Let x be the number of attempts. Now the information provided may be written below.

Cost of Plan A: $50 + 25x$
Cost of Plan B: $30 + 45x$

According to the given condition model is $\text{Cost with Plan A} < \text{Cost with plan B}$.

In algebraic form: $50 + 25x < 30 + 45x$

$20 + 25x < 45x$ subtract 30
$20 < 20x$ subtract $25x$
$1 < x$ Divide 20

So, if you plan to take more than one attempt, Plan A is less expensive.

Example: The labeling on the insulin vial specifies a required storage temperature range between 36^oF and 46^oF. What is the equivalent temperature range on the Celsius scale?

Solution: As we know the relationship between degrees Fahrenheit (F) and degrees Celsius (C) is $F = \frac{9}{5}C + 32$. According to the suitable conditions on the insulin bottle, we have $36 < F < 46$

By using the relation $F = \frac{9}{5}C + 32$, we have

$36 < \frac{9}{5}C + 32 < 46$
$4 < \frac{9}{5}C < 14$ subtract 32
$\frac{5}{9}(4) < C < \frac{5}{9}(14)$ Multiply $\frac{5}{9}$
$2.22 < C < 7.78$

So, the insulin should be stored at a temperature between 2^oC and 8^oC degree.

8.6 Self-Assessment Questions

1. Fill in the blanks with a suitable inequality sign.

   i. If $u < 7$, then $u - 4$ ____ 3.

   ii. If $u \leq 7$, then $3u$____ 21.

   iii. If $u \geq 5$, then $-2u$ ____ -10.

   iv. If $u < -7$, then $-u$ ____ 7.

   v. If $u < 2$, then $u + 4$ ____ 6.

2. Fill in the blanks expressing the given absolute value inequality in interval notation.

   i. The inequality $|u| \leq 1$ has the solution in the interval _____.

   ii. The inequality $|u| \geq 1$ has the solution as the union of two intervals _____.

3. Fill in the blanks expressing the given problem in absolute value inequality.

   i. The set of all real numbers that are within 2 units from 0 can be defined using the absolute value inequality. $|u|$ _____.

   ii. The set of all real numbers that are more than 2 units away from 0 can also be defined using the absolute value inequality $|u|$ _____.

4. Answer true or false. Also, justify by the examples in favor and not.

   i. The inequality $u(u + 1) > 0$ follows that u is always positive?

   ii. The inequality $u(u + 1) > 3$ follows that $u > 3$?

In Problems 5–22 Solve the following inequalities. Write down the solution interval notation and represent the solution interval on the real line.

5. $2u \leq 5$
6. $3u - 7 > 5$
7. $-3u \geq 15$
8. $3u + 11 < 7$
9. $2u - 2 \leq 0$
10. $6 - u \geq 3$
11. $4u - 7 < 8 + 9u$
12. $0 < 9 - 2u$
13. $3u - 4 \geq 1 + 8u$
14. $-1 \leq 2u - 7 < 5$
15. $\frac{1}{2}u - \frac{2}{3} < \frac{1}{3}$
16. $\frac{1}{6} < \frac{3u-12}{3} \leq \frac{2}{3}$
17. $-3 < 3u + 7 \leq \frac{1}{2}$
18. $-8 < 2u - 6 \leq 4$
19. $\frac{1}{3}u + 2 < -\frac{1}{6}u - 4$
20. $5 < u + 5 < 12$
21. $-2 < 8 - 2u \leq -1$
22. $3(2u + 7) \geq 4u + 14$

In Problems 23-40 Solve the following non-linear inequalities. Write down the solution interval notation and represent the solution interval on the real line.

23. $(u + 4)(u - 5) < 0$
24. $(u - 6)(u + 5) \geq 0$
25. $u(3u + 7) \geq 0$
26. $u(3 - 2u) \leq 0$
27. $u^2 - 2u - 3 \leq 0$
28. $u^2 + 5u + 6 > 0$
29. $u^2 < 3u + 18$
30. $u^2 \geq -u + 6$
31. $2u^2 + 7u \geq 0$
32. $2u - 3u^2 \leq 0$
33. $u^2 < 16$
34. $u^2 \geq 4$
35. $6u^2 + 3u \geq 4u^2 + 2$
36. $5u^2 - 5u \geq 4u^2 + 2u + 2$
37. $(u + 3)(u - 2)(u - 4) \leq 0$
38. $(u - 6)(u - 3)(u + 2) > 0$
39. $(u - 5)(u + 3)^2 < 0$
40. $(u + 4)^2(u + 2) > 0$

In Problems 41-54 Solve the following inequalities. Write down the solution interval notation and represent the solution interval on the real line.

41. $\frac{u-3}{u+2} \geq 0$
42. $\frac{2u+6}{u-1} < 0$
43. $\frac{u+1}{u+2} > 3$
44. $\frac{2u+1}{u-4} > \frac{1}{5}$
45. $\frac{u-5}{2u+1} \geq \frac{1}{3}$
46. $\frac{4-u}{4+u} \leq 1$
47. $\frac{u}{9} > \frac{1}{u}$
48. $\frac{u}{3u+3} > u$
49. $2 + \frac{1}{u-1} \leq \frac{2}{u}$
50. $\frac{4}{u} - \frac{3}{u-1} \leq -1$
51. $\frac{-5}{u+1} - 4 \leq -\frac{u}{2}$
52. $\frac{u+3}{u+2} > \frac{u-2}{u-1}$
53. $1 \leq 2u - 8 \leq 2$
54. $-1 \leq 2(-4u - 7) \leq 6$

In Problems 55–66 Solve the inequalities involving absolute values. Write down the solution using interval notation and create a graph representing the solution.

55. $|4u| < 16$
56. $|3u| \geq 4$
57. $-\frac{1}{3}|u| \geq 2$
58. $|7u - 6| < 8$
59. $|3u - 4| \geq 5$
60. $|5u + 3| < 5$
61. $\left|\frac{u-5}{3}\right| < 4$
62. $\left|\frac{2u+4}{2}\right| \geq 7$
63. $|2u - 1| < 0.003$
64. $5 - |3u + 4| \leq 3$
65. $8 - |3u - 1| \geq 6$
66. $2|u + 2| + 5 > |u + 2| + 4$

In Problems 67–71 Express each of the following sets as an absolute value inequality.

67. All reals u at least 2 units from 5.

68. All reals u at most 3 units from 1.

69. All reals u greater than 4 units from 0.

70. All reals u less than 5 units from 2.

71. All reals u more than 1 units from 0.

Unit–9

LOGICAL THINKING

Written by: Dr. Irfan Younas

Reviewed by: Dr. Muhammad Asad Zaigham

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9.1 Introduction

The word "logic" has its roots in the Greek word "logos," which means reasoning. In essence, logical thinking refers to the practice of sound reasoning or making good arguments. Logical thinkers gather data, examine, evaluate, and record events, responses, and feedback before making judgments based on their findings. Based on the information they get, they may defend the tactics, choices, and actions they choose. Furthermore, rational thinking calls for the clarification of presumptions and the maximum avoidance of prejudices. These days, we respect employees who can reason or think logically since their decisions are supported by solid evidence.

9.2 Objectives

At the end of this unit students will be able to:

- Solve practical problems logically
- Survive in the modern world using logic
- Understand propositions and truth values
- Applications of logic
- Improve their ability to use quantitative reasoning in decision-making

9.3 Inductive and Deductive Logic

In our daily lives, we tend to draw conclusions and form opinions based on limited information. For example, if someone has a reaction to a penicillin injection, they may conclude that they are allergic to penicillin. This type of reasoning is called induction. Conversely, we can draw conclusions based on well-established facts or premises, which is known as deduction.

The idea of proof is central to mathematics. A series of logical arguments that demonstrate the veracity of a theorem or assertion constitute a mathematical proof. If a theorem or assertion has a solid and reliable proof, it is said to be proved. While induction is frequently used to find theorems, mathematical proofs are based on deduction.

Inductive reasoning is particularly useful in natural sciences where we draw conclusions from repeated experiments or observations. The process of induction is helpful when we are struggling to determine if a particular theorem or mathematical rule applies. By applying specific test cases, we can determine if a rule is true. Although these test cases can never provide definitive proof, they can sometimes be sufficient to establish the rule's veracity. However, if any of the test cases fail, the rule may be false. In fact, much of our knowledge is based on induction.

Example: Inductive approach
Test the rule inductively: For all a, b and c, $c-(b-a)=a-(b-c)$.

Solution: Consider some test cases.

Does $7-(-3-2)=2-(-3-7)$? The answer is yes.

Does $-7.42-(11.1-9.5)=9.5-(11.1-(-7.42))$? The answer is yes.

Does $-4.01-(\frac{1}{3}-\frac{2}{5})=\frac{2}{5}-(\frac{1}{3}-(-4.01))$? The answer is yes.

Each of the three test cases (integers, decimals, and mixed fractions) follows the same rule

$$c-(b-a)=a-(b-c).$$

On the basis of the evidence gathered; we are highly confident that the rule in question is accurate. However, our confidence level could be further enhanced by conducting additional tests that validate the rule's effectiveness.

9.4 Propositions and Truth Values

A premise in logic is a claim that may or may not be true. It serves to bolster a case that logically draws a conclusion that is either accurate or incorrect. A minimum of two declarative sentences, referred to as premises, and one additional declarative sentence, referred to as the conclusion, are necessary for an argument. A simple argument consists of one conclusion and two premises. In symbolic logic, more complicated explanations may use a set of rules to link several premises to a single conclusion. There are times when premises are not mentioned clearly. These are called "missing premises." Let us examine the logic behind the claim that "Socrates is mortal because all men are mortal." The premise behind this is that Socrates is a man. "Since Socrates is a man and all men are mortal, Socrates must also be mortal." This is the case's complete rationale. The propositions "All men are mortal" and "Socrates is a man" imply that "Socrates is mortal."

The validity of an argument in philosophy or any field depends on the accuracy and truth of its premise or premises. For instance, let's consider an argument with a false premise: "All men are politicians." Although the minor premise "Qazi Faez Isa is a man" is true, the conclusion "Qazi Faez Isa is a politician" is false. In this example, the false premise is "all women are Republicans," and the true premise is "Hilary Clinton is a woman." Hence, the conclusion is false.

Thoughts that are based on logic often use statements. There are two possible answers to the question "What is a proposition?" Aristotle thought that statements could only be true or wrong, and not a third way. Examples of propositions include premises and conclusions. This is particularly true for mathematics and other exact sciences. Two types of logical reasoning are typically used based on propositions derived from a limited number of experiments and observations.

9.4.1 Negation (Opposite)

Negation is the logical antithesis of a proposition. For instance, 1-6 = 3 has the negative 1-6≠3, and the negation of Nehal driving a car is that Nehal is not driving a car. When a statement is represented by a letter like "p ", the negation of that proposition would be "not p ", which is represented by "~p "). Since negatives have the same structure as a whole sentence, they can also be true or untrue. A proposition's negation must be false (F) if it is true (T), and vice versa.

To illustrate these facts, we may create a truth table. Rows in a truth table reflect all possible combinations of truth values. The potential truth values for a statement p and its negation, not p, are displayed in the truth table that follows.

The truth table has only two rows since there are only two possible outcomes: true or false.

p	$\sim p$
F	T
T	F

We could make a truth table to show these facts. A truth table's rows show every conceivable pairing of truth values. The truth table that follows shows the possible truth values for a statement p and its negation, not p. Since there are only two possible outcomes—true or false—the truth table only has two entries.

Definitions: A truth table shows all the possible truth values for a set of statements.

Example: Negation

Find out what the plan doesn't say. The head judge of Pakistan is Qazi Faez Esa. If it's not true, is Qazi Faez Esa really the Chief Justice of Pakistan?

Solution: The statement is rejected since Qazi Faez Esa does not hold the status of Chief Justice of Pakistan. Preceding the preceding assertion, Qazi Faez Esa is the Chief Justice of Pakistan unless this negation is incorrect.

9.4.2 Double Negation

The negation "not" has the same truth value as the proposition, as has already been demonstrated. This indicates that, in terms of truth value, the supplied assertion and the double negation " not" are equal. To demonstrate this, we can use a truth table. The first column shows the two possible truth values for the statement that was delivered. There are two additional columns that show the matching truth values for "not" and "not not."

p	$\sim p$	$\sim\sim p$
F	T	F
T	F	T

9.4.3 Logical Connectors

Logical connectors such as 'and', 'or', and 'if...then' are frequently used to combine propositions. Let's consider the following two propositions as an example:

p = The road was difficult.
q = I reached early.

If we join these propositions using 'and', the new proposition would be "The road was difficult and I reached early". On the other hand, if we use 'or', the statement would be "The road was difficult and I reached early". It's common to use such statements in everyday conversations, but we need to analyze them carefully.

9.4.4 Conjunction (And)

A conjunction is a sentence that uses the word "and" to join two ideas together. The conjunction "p and q" stands for "p and q" if p and q are two propositions. If both p and q are true, then the combination $p \wedge q$ is also true. Saying "the road was difficult and I reached early" is only true if the road really was difficult, and you did make it there early.

To create a truth table for the logical conjunction of $p \wedge q$, we need to examine every possible combination of truth values for p and q separately. Since each proposition can result in either true or false, there are four possible scenarios in total $(2 \times 2 = 4)$. These four scenarios are represented as four rows in the truth table.

Truth Table for $p \wedge q$		
p	q	$p \wedge q$
F	F	F
F	T	F
T	F	F
T	T	T

Take note that only the first example in the table, when each of the two premises is true, makes the statement true.

Example: Find out how true each of these two claims is.

a. Islamabad is the capital of Pakistan, while Chennai is the capital of India
b. Islamabad hosts the Pakistani capital, while Lahore serves as the Punjabi capital

Solution:

a. Here, we have two propositions. The first proposition is true, as the capital of Pakistan is indeed Islamabad. However, the second statement is false, as the capital of India is not Chennai. Therefore, the conjunction of these two statements is false.

b. Here, we also have two distinct statements. Both statements are true, as the capital of Pakistan is indeed Islamabad, and the capital of Punjab is Lahore. Therefore, their conjunction is also true.

Example: Triple Conjunction

Create the truth table for the conjunction of three individual propositions p, q and r. Also, determine the conditions under which the conjunction is true.

Solution: We must take into account every scenario involving the three propositions—p, q, and r, in order to create a truth table for their combination. There are four possible outcomes for the combination of p and q, as we already know. Proposition r can be regarded as true or untrue in each of these four scenarios, giving rise to eight potential scenarios for the combination of p, q, and r. For any combination of p, q, and r truth values, the conjunction's truth value is displayed in the truth table that follows. Remember that each of these things can happen twice, once when r is true and once when it is wrong. In this way, we can find the situations in which the combination of p, q, and r is correct.

Truth Table for Conjunction $p \wedge q \wedge r$

p	q	r	$p \wedge q \wedge r$
F	F	F	F
F	F	T	F
F	T	F	F
F	T	T	F
T	F	F	F
T	F	T	F
T	T	F	F
T	T	T	T

There is only one true combination between p, q, and r, which can be seen in the last row:

Activity: In a truth table containing the conjunction p and q and r and s, given four propositions, how many rows should there be? For what situations does the conjunction hold true?

9.4.5 Disjunction (Or)

Depending on the context, the connection could signify one of two things. For instance, it's likely that health insurance coverage only covers one of the two scenarios of illness or injury if it says that hospitalization is covered. In this context, "either" or "both" refers to the inclusive or option. Conversely, you are often only allowed to enroll in one of these programs when a university gives you the option between pure mathematics and applied mathematics. The phrase "exclusive or" describes this situation, meaning" one or the other". In logic, we take things at face value and presume inclusion unless told differently.

In real life, however, the context dictates whether a statement is inclusive or not.

Definition: A disjunction is a compound sentence that employs the connective 'or'. When two propositions, p and q, the disjunction "p or q" is represented by $p \vee q$. Since the 'or' is interpreted as inclusive, the disjunction $p \vee q$ is true if and only if each of the component propositions is true. Conversely, a disjunction $p \vee q$ is only untrue if and only if each of the component propositions is incorrect.

The four scenarios arose by the connector 'or' are represented by the following truth table.

Truth Table for the Disjunction $p \vee q$

p	q	$p \vee q$
F	F	F
F	T	T
T	F	T
T	T	T

Example: Smart Men

Think about the statement man can fly or cows can provide milk. Is this statement really true?

Solution: The claim that men can fly and that cows can produce milk are the two separate assertions that make up the statement. The second claim is unquestionably true, whereas the first is untrue. It is true that cows can produce milk, and men can fly.

9.4.6 Conditional (If …Then)

The phrase "if... then" is frequently used to connect assertions, as in the case of the sentence "If all cricketers are athletes, then cricketer Shadab is an athlete." Because they imply that something is true only in the event that another item is true, these statements are known as conditional propositions. If there are two propositions, p and q, then $p \rightarrow q$ represents the conditional if p then q. In this case, propositions p and q are referred to as the hypothesis and therefore, respectively, and the conclusion.

Let's consider an example to discuss the truth table of the statement "if...then". Suppose that Rizwan, a player on the Pakistan cricket team, claims that if he is the captain, then Pakistan will win the World Cup. Here, hypothesis p = Rizwan is captain and q = Pakistan will win the World Cup. The following four possible truth values arise for "if p, then q" since each proposition can be true or false.

1. p and q both true. In this case, Rizwan was captain (p true) and the Pakistan won the World Cup (q true). Rizwan did work hard to win the series of World Cup matches and his claim denoted by $p \rightarrow q$ was true.

2. p true and q false. In this case, Rizwan was captain, but Pakistan lost the World Cup. Because things did not turn out as claimed, If Rizwan was the captain, then Pakistan won the World Cup, which was false.

3. p false and q true. In the scenario where Rizwan was not chosen as the captain, but Pakistan still managed to win the World Cup tournament, the conditional statement asserts that Rizwan should have been selected as the captain before the tournament. Since he was not chosen, he did not break any commitment. Therefore, regardless of the outcome, logically, Rizwan's claim should be considered true in this case.

4. p and q both false. In the scenario where Rizwan was not chosen as the captain and Pakistan did not emerge victorious in the World Cup tournament, it is clear that he did not breach his commitment since he was not in charge. Therefore, his claim in this case is valid, just like in the previous scenario.

Truth Table for Conditional if $p \rightarrow q$		
P	Q	$p \rightarrow q$
F	F	T
F	T	T
T	F	F
T	T	T

Briefly, the statement "if p, then q" is true except when p is true, and q is false.

Definition: A conditional proposition, also known as implication, takes the form of "if p, then q". The statement consists of two parts: the hypothesis (p) and the conclusion (q). In all situations, except for the case where p is true and q is false, the conditional statement is considered true.

Example: Conditional Truths
Evaluate the truth of the statement if $3 + 3 = 5$, then $4 + 4 = 4$.

Solution: "If p, then q" is stated, with p being "3+3=5" and q being "4+4=4". Given that both p and q are untrue. The conditional sentence "if p then q" is therefore true anytime p is false, independent of what q says, according to the laws of logic. As a result, it is true that "if 3+3=5, then 4+4=4".

9.5 Alternative Statements of Conditionals

In our daily life, Conditional statements may not always occur in the standard form of "if p, then q". Restating the assertions to conform to the conventional form might be helpful in these situations. For instance, the statement "I am going home if there is no exam today" can be restated as "If there is no exam today, then I am going home". Similarly, the statement "frequent use of alcohol can lead to affect liver" can be restated as " If alcohol is used frequently, it can affect the liver.".

Two common ways to express conditionals are by using the words "necessary" and "sufficient". For example, the conditional "If a person is alive, then he/she will be breathing" is true. We can restate this conditional as "Breathing is necessary for a person to be alive". It's crucial to understand that this phrase does not suggest that breathing is the only requirement for survival. Rather, breathing is just one of several needs, along with food and beating. Generally speaking, the statement "q is necessary for p" is identical to any true conditional of the form "if p, then q".

Now consider another example; If you are in Lahore, then you are in Punjab. We can restate this statement as Being in Lahore is sufficient for being in Punjab because Lahore is a city in Punjab. Note, however, that being in Lahore is not necessary for being in Punjab, because you could also be in other places in the Punjab. More generally, any true statement of the form if p, then q is equivalent to the statement p is sufficient for q.

The most familiar alternative expressions for "if p, then q" are as follows:

q is necessary for p	q if p	q whenever p
p is sufficient for q	p will lead to q	p implies q

Example: Rephrasing Conditionals

Restate each of the following conditionals in the form if p, then q.

a) An increase in terrorism will destroy Pakistan's economy.

b) A green shirt is sufficient to indicate Pakistani players.

c) A daily exercise is necessary for good health.

Solution:

a) It can be expressed that if terrorism increases, then Pakistan’s economy will be destroyed.

b) It can be written that if a player wore a green shirt, then he is Pakistani.

c) It is equivalent to if a person is in good health, then the person exercises daily.

9.5.1 Converse, Inverse and Contrapositive

When using a conjunction or disjunction, the order of the propositions is not important. For instance, "p and q" has the same meaning as "q and p", and "p or q" is equivalent to "q or p". The reverse, on the other hand, is a distinct assertion that results from rearranging the propositions in a conditional statement. The definitions of the converse and two more forms of a conditional statement

"if p, then q" are compiled in the table below.

Name	How to read	Symbolic representation	Example
Conditional	if p, then q	$p \to q$	If you are in a green shirt, then you are a Pakistani.
Converse	if q, then p	$q \to p$	If you are a Pakistani, then you are in a green shirt.
Inverse	if not p, then not q	$\sim p \to \sim q$	If you are not in a green shirt, then you are not a Pakistani.
Contrapositive	if not q, then not p	$\sim q \to \sim p$	If you are not a Pakistani, then you are not in a green shirt.

By using the principle that a conditional statement is false only when the hypothesis is true and the conclusion is false, we may get the truth values for the converse, inverse, and contrapositive.

p	q	$\sim p$	$\sim q$	$p \to q$	$q \to p$ (converse)	$\sim p \to \sim q$ (inverse)	$\sim q \to \sim p$ (contrapositive)
F	F	T	T	T	T	T	T
F	T	T	F	T	F	F	T
T	F	F	T	F	T	T	F
T	T	F	F	T	T	T	T

The truth values for a conditional statement "if p, then q" are the same as the truth values for its contrapositive, which is a crucial distinction to make. This indicates that there is a logical equivalence between a conditional statement and its contrapositive. Both are true if the first is, and vice versa. The opposite and inverse are logically equal, as the table also demonstrates.

9.6 Logical Equivalence

If the truth values of two propositions are the same, then they are logically comparable. In the event that one is true, the other must also be true, and vice versa.

Example: Logical Equivalence

Rewrite the statement "If a creature is a whale, then it is a mammal" to include its converse, inverse, and contrapositive. Therefore, consider each of the following propositions and rank their validity. Together, determine which assertions make sense.

Solution: This sentence is a conditional "if p, then q" in which "a creature is a whale" is represented by antecedent p and "a creature is a mammal" by consequent q. We may look at the converse, inverse, and contrapositive to have a better understanding of the link between the antecedent and consequent.

- Counterargument: "If a creature is a mammal, then it is a whale" is false because most mammals are not whales.
- On the other hand, the statement "if a creature is not a whale, then it is not a mammal" is false as well because dogs and other mammals are not whales.
- The contrapositive "if a creature is not a mammal, then it is not a whale" is true since all whales are mammals.

Fascinatingly, the original claim and its contrapositive share the same truth value and are logically equal. The converse and inverse also have the same truth value and are logically equivalent.

9.6.1 Types of Compound Statements

A compound statement that holds true for all possible values of the propositions involved in it is called a **tautology**. On the other hand, a statement that is always false is called an **absurdity** or a contradiction. Lastly, a statement that can either be true or false depending on the integrity values of the ideas involved is known as **contingency**.

Example: Logical Equivalence

Construct the truth tables to identify which of the following is tautology or absurdity or contingency.

a) $if\ (not\ (if\ p, then\ q)), then\ p$

b) $q\ or\ (not\ q\ or\ p)$

9.7 Self-Assessment Questions

1. In the following questions, you'll be presented with multiple options. Choose the best one and explain why in a few brief sentences.

(i) The assertion that "Mathematics is fun" is
   a) a fallacy.
   b) a proposition.
   c) a conditional.

(ii) The truth value of a premise p can help us figure out the truth value of a statement x.
   a) conjunction
   b) negation
   c) truth table

(iii) Which of the following statements is conditional?
   a) x and y
   b) if x, then y
   c) x or y

(iv) How many rows are there in a table containing disjunction of three propositions, that is, $p\ or\ q\ or\ r$?
   a) 3
   b) 8
   c) 4

(v) The disjunction of two propositions $p \vee q$ is true, if
   a) q is true.
   b) any one or both are true.
   c) p is true.

(vi) Assume, for the sake of argument, that q is true while p is untrue. In your opinion, which of these claims is not true?
   a) $p \wedge q$
   b) $p \vee q$
   c) $p \rightarrow q$

(vii) The statement "If he is a Pakistani, then he is an Asian" can be restated as
   a) Being Asian is enough to be called a Pakistani.
   b) One must be Asian in order to be a Pakistani.
   c) All Asians are Pakistanis.

(viii) It is logically identical to say that "If a person is breathing, then the person must be alive."
   a) If a person is alive, then the person must be breathing.
   b) If a person does not breathe, then the person is not alive.
   c) If a person is not alive, then the person must not be breathing.

   Two assertions are logically equivalent if
   a) They are both true.
   b) They represent the identical thing.
   c) They hold the same identical values.

(ix) Consider a conditional “You have reached to participate in the game if you want to win”. Compare it with the conditional “if p, then q ”, then q must be
   a) You have reached to participate in the game.
   b) You want to win.
   c) You have reached to participate in the game if you want to win.

2. What does a promise mean? Here are some examples of propositions. Please explain why each one is a proposition.

3. Define the terms conjunction, disjunction, and conditional. Additionally, could you provide an example of each term in a sentence?

4. What is the meaning of negation in propositional logic? Can you provide an example of a proposition and its negation?

5. How are inclusive and exclusive or different from each other? Could you give an example of each?

6. Write down the truths for p and q, p or q, and if p then q. List all the meanings of truth.

7. How to create a conditional statement's converse, contrapositive, and opposite statements, as well as their corresponding truth tables. Next, determine which of the two statements follows reasoning from the original.

Identify whether the given statements are propositions or not. Additionally, you need to provide an explanation for your answer.

8. Every person is an actor.
9. The food is tasty.
10. Back to the home.
11. Some troglodytes are hippophobic.
12. What are you saying?
13. The chairman is a famous mathematician.

Rewrite the proposition in negation form and determine truth values.

14. Pakistan is in South Asia.
15. The brand Lewis was initially from England.
16. Nauman Ijaz is a Pakistani actor.
17. Ali is going to have dinner.

Explain the given sentence that has more than one negative, and then answer the question that comes after each statement.

18. Maira did not decline the offer to buy a new car. Did Maira buy a new car?
19. The senator voted against the bill that was meant to hurt Muslims. Did the Senator vote for the bill that was against Islam?
20. The mayor denies that he opposes the construction of a new secretariat. Does the Mayor support or oppose the construction of a new secretariat?

Use the arguments p, q, r, and s to make a truth table for the given sentence.

21. q and s
22. p and r
23. p and q and s
24. p and r and q and s

25. r or s

26. p or r

27. p and (not p)

28. q or (not q)

29. p or q or r

30. p or (not p) or q

31. if r, then s

32. if q, then p

Identify the truth values of p and q, and determine if the overall proposition is true or false. Provide a brief explanation for your conclusion.

33. Cucumbers are vegetables and apples are fruit.

34. $13 + 6 = 19$ and $2 \times 6 = 10$.

35. The Ravi River goes through Punjab and the Sindh River goes through Gilgit-Baltistan.

36. Khawaja Khurshid Anwar was a composer and Nusrat Fateh Ali Khan was a singer.

37. Not all animals are goats and not all men are politicians.

State whether "or" is used inclusively or exclusively in the following propositions.

38. I will spend my next vacation in Lahore or Karachi.

39. The oil change is good for three months or five thousand miles.

40. I'd like to play cricket or football while I'm in New York.

41. The insurance policy covers fire or theft.

Identify the truth values of p or q, and determine if the overall proposition is true or false. Provide a brief explanation for your conclusion.

42. You may eat cucumbers as either a vegetable or a fruit.

43. $4 \times 5 = 20$ or $4 + 5 = 9$.

44. The Sindh River is in Pakistan and Lahore is in Europe.

45. Trees can walk or rocks can run.

46. Pakistan is a country or Lahore is a continent.

Identify the truth values of the hypothesis and conclusion in the given statements and determine if the overall proposition is true or false. Provide a brief explanation for your conclusion.

47. Parrots are birds if they can fly.

48. Delhi is in India and Lahore is in Pakistan.

49. New Delhi is in Afghanistan and Lahore is in England.

50. If Lahore is in Bangladesh, then Chicago is in the United States.

51. If cows can fly, then fish can live without water.

52. If $3 \times 7 = 21$, then $3 + 7 = 11$.

53. If parrots are birds, then parrots can fly.

Write the following sentences in the form if p, then q. Find p and q without any trouble.

54. A resident of Karachi is a resident of Pakistan.

55. Daily exercise is a sufficient condition for being healthy.

56. Being bald is sufficient for being male.

57. Whenever it rains, I get wet.

58. There is no doubt that she is smart if she is an art expert.

Find the opposite, inverse, and contrapositive of each of the statements below and say which pairs are the same.

59. If Ali works in AIOU, then he owns a laptop.

60. If the weather is cloudy, then it is cold outside.

61. If the glaciers melt, then the water level will rise in the rivers.

62. The lights are on when I wake up.

63. If Sara lives in Lahore, then she lives in Pakistan.

64. If the heater is on, then it is cold outside.

Write the following quotations in necessary as well as sufficient forms.

65. You can do it if you think you can. Says Tupac Shakur

66. If it's true that our species is the only one in the universe, then the universe didn't reach very high. —The great George Carlin

67. We will be a country gone under if we ever forget that we are One Country Under God. —Ronald Reagan

68. This means that if you need both hands to do something, then your brain should also be involved. —How Ellen DeGeneres

To find out if two claims p, q, and r are logically equivalent, make a truth table for each pair of them.

69. $\sim (p \wedge q);\ (\sim p) \vee (\sim q)$

70. $\sim (p \vee q);\ (\sim p) \wedge (\sim q)$

71. $\sim (p \wedge q);\ (\sim p) \wedge (\sim q)$

72. $\sim (p \vee q);\ (\sim p) \vee (\sim q)$

73. $(p \wedge q) \vee r;\ (p \vee r) \wedge (p \vee q)$

74. $(p \vee q) \wedge r;\ (p \wedge r) \vee (q \wedge r)$

75. $(p \wedge q) \wedge r; p \wedge (q \wedge r)$

Rewrite each of the following logical statements into symbolic notation, then construct a truth table to determine whether each of the following is a tautology, an absurdity, or a contingency.

76. $(if\ p, then\ not\ p\ )\ or\ (if\ p, then\ q)$

77. $if\ (\ p\ and\ {\sim}p), then\ r$

78. $if\ p, then\ (p\ or\ q$

79. $not\ q\ and\ (if\ (if\ p, then\ q), then\ not\ p)$

80. $if\ P, then\ (if\ q, then\ p)$

81. $q\ and\ not\ q$

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